Answer on Question #48567, Math, Trigonometry

In ΔABC, A = 20, B = 80, C = 80. D lies on AC such that angle DBC = 70. E lies on AB such that ECB = 50. Using Trigonometric Version of Ceva's Theorem deduce that angle BDE = 10. All measures are in degree.

Solution.
By applying Trigonometric Version of Ceva's Theorem for ΔDEC we get next equality:

$$\frac{\sin\angle EDB}{\sin\angle BDC}\cdot\frac{\sin\angle DCB}{\sin\angle BCE}\cdot\frac{\sin\angle CEB}{\sin\angle BED}=1.$$

From the task we know that $\angle CEB=\angle BCE=50$, and $\angle BDC=30$, $\angle DCB=80$. The angle $\angle BED=170-\angle EDB$, which could be found from ΔBED. Than we substitute this angles in the equality and obtain the new one:

$$\frac{\sin\angle EDB}{\sin 30}\cdot\frac{\sin 80}{\sin(170-\angle EDB)}=1.$$

Hence, $2\sin 80\cdot\sin\angle EDB=\sin(170-\angle EDB)$, transforming this equality we obtain:

$$\cos(80-\angle EDB)-\cos(80+\angle EDB)=\sin(170-\angle EDB) \Rightarrow$$

$$\sin(10+\angle EDB)-\sin(10-\angle EDB)=\sin(10+\angle EDB).$$

Finally, $\sin(10-\angle EDB)=0$ hence $\angle EDB=10$.

Answer: EDB=10.