Answer on Question #48475 – Math – Calculus

A freshly brewed cup of coffee has temperature 95°C in a 20°C room. When its temperature is 69°C, it is cooling at a rate of 1°C per minute. When does this occur? (Round your answer to two decimal places.)

$T_0 = 95^0 C$

$T_r = 20^0 C$

$T_1 = 69^0 C$

$\frac{\Delta T}{\Delta t} = 1^0 \frac{C}{\min}$

$t - ?$

Solution.

The rate of temperature change is proportional to the difference between temperature T of the body and room temperature :

$$\frac{dT}{dt} = -k(T - T_r).$$

The solution of this equation with initial condition $T(0) = T_0$ is

$$T(t) = T_r + (T_0 - T_r)e^{-kt}.$$

Cooling rate at the moment t is $k \cdot (T(t) - T_r)$.

According to the text, we can write that $\begin{cases} T_1 = T_r + (T_0 - T_r)e^{-kt} \\ \frac{\Delta T}{\Delta t} = k \cdot (T_1 - T_r) \end{cases}$.

Solving the system of the equations, one can obtain the unknown time:

$$\boxed{t = \frac{T_1 - T_r}{\frac{\Delta T}{\Delta t}} \ln \frac{T_0 - T_r}{T_1 - T_r}}.$$

Let check the dimension: $[t] = \frac{{}^0C}{\frac{{}^0C}{\min}} = \min$.

Let evaluate the quantity: $t = \frac{69 - 20}{1} \cdot \ln \frac{95 - 20}{69 - 20} = 20.9(\min)$.

Answer: 21 min.