Answer on Question #48124 – Math – Abstract Algebra:

Which of the following subsets are subgroups of D_{12}? Justify your answer.
i) $\{x, y, xy, y^2, y^3, e\}$

ii) $\{xy, xy^2, y^2, e\}$

iii) $\{x, y^3, xy^3, e\}$

Solution.

$$D_{12} =< x, y \mid x^2 = y^{12} = (xy)^2 = e >;$$

i) $y^2 \cdot y^3 = y^5$;

$$ord(y^5) = 12;$$
$$ord(x) = ord(xy) = 2;$$
$$y^5 \neq y, y^5 \neq y^2, y^5 \neq y^3, y^5 \neq e;$$

Hence, $y^5 \notin \{x, y, xy, y^2, y^3, e\}$. So, it is not a subgroup.

ii) $y^2 \cdot y^2 = y^4$;

$$ord(y^4) = 3;$$
$$ord(xy) = 2;$$
$$xy^2 = y^4 \Longrightarrow x = y^2 - \text{contradiction};$$

Hence, $y^4 \notin \{xy, xy^2, y^2, e\}$. So, it is not a subgroup.

iii) $y^3 \cdot y^3 = y^6$;

$$y^6 = x \Longrightarrow (xy)^2 = y^{14} = e - \text{contradiction};$$
$$y^6 = y^3 \Longrightarrow y^3 = e - \text{contradiction};$$
$$y^6 = xy^3 \Longrightarrow x = y^3 - \text{contradiction}.$$

Hence, $y^6 \notin \{x, y^3, xy^3, e\}$. So it is not a subgroup.