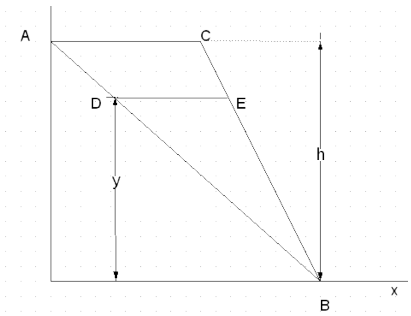
Answer on Question #48087 – Math – Geometry

How to prove the volume formula of a pyramid is $\frac{1}{3}h(area\ of\ the\ surface)$?

Solution



On the left, ACB is a section of a pyramid. AC is a line through the base, and represents one dimension of the area, A, of the base. DE is any section through the pyramid at height y, and is parallel to the base, and represents an area A_i, h is the height of the pyramid.

Because the area section at DE, A_i, is proportional to the area of the base of the pyramid, A (at AC) and these areas, A and A_i, are proportional to their heights squared, then:

$$\frac{A_i}{A} = \frac{y^2}{h^2}.$$

So, A_i is:

$$A_i = A\frac{y^2}{h^2}.$$

As each component of the total volume is:

$$dV = A_i dy.$$

Substituting the formula for the area:

$$dV = A\frac{y^2}{h^2}dy.$$

We now have the means of integrating to get the volume, V, between the limits 0 and h:

$$V = \int_0^h A\frac{y^2}{h^2}dy.$$

And by integrating:

$$V = \left(A\frac{y^3}{3h^2}\right)_0^h = \frac{1}{3}Ah.$$