Answer on Question #48027 – Math – Trigonometry

given: sinΘ = 4/5 cosΦ =12/13 find sin (Θ + Φ)

Solution:

$\sin\theta = \frac{4}{5}$

$\cos\Phi = \frac{12}{13}$

Hence, $\Theta \in [0^\circ; 180^\circ]$, because sine of Θ is positive; $\Phi \in [0^\circ; 90^\circ]$ or $\Phi \in [270^\circ; 360^\circ]$, because cosine of Φ is positive.

Relationship between the sine and the cosine (Pythagorean identity):

$$\sin^2\theta + \cos^2\theta = 1$$

A. Suppose that $\Theta \in [0^\circ; 90^\circ]$ and $\Phi \in [0^\circ; 90^\circ]$.

$$\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \frac{3}{5}$$

$$\sin^2\Phi + \cos^2\Phi = 1$$

$$\sin\Phi = \sqrt{1 - \cos^2\Phi} = \sqrt{1 - \left(\frac{12}{13}\right)^2} = \frac{5}{13}$$

Sine of sum:

$\sin(\theta + \Phi) = \sin\theta\cos\Phi + \cos\theta\sin\Phi = \frac{4}{5}\cdot\frac{12}{13} + \frac{3}{5}\cdot\frac{5}{13} = \frac{63}{65}.$

B. Suppose that angles $\Theta \in [0^\circ; 90^\circ]$ and $\Phi \in [270^\circ; 360^\circ]$.

$$\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \frac{3}{5}$$

$$\sin^2\Phi + \cos^2\Phi = 1$$

$$\sin\Phi = -\sqrt{1 - \cos^2\Phi} = -\sqrt{1 - \left(\frac{12}{13}\right)^2} = -\frac{5}{13}$$

Sine of sum:

$\sin(\theta + \Phi) = \sin\theta\cos\Phi + \cos\theta\sin\Phi = \frac{4}{5}\cdot\frac{12}{13} - \frac{3}{5}\cdot\frac{5}{13} = \frac{33}{65}.$

C. Suppose that $\Theta \in [90^\circ; 180^\circ]$ and $\Phi \in [0^\circ; 90^\circ]$.

$$\cos\theta = -\sqrt{1 - \sin^2\theta} = -\sqrt{1 - \left(\frac{4}{5}\right)^2} = -\frac{3}{5}$$

$$\sin^2\Phi + \cos^2\Phi = 1$$

$$\sin\Phi = \sqrt{1 - \cos^2\Phi} = \sqrt{1 - \left(\frac{12}{13}\right)^2} = \frac{5}{13}$$

Sine of sum:

$\sin(\theta + \Phi) = \sin\theta\cos\Phi + \cos\theta\sin\Phi = \frac{4}{5}\cdot\frac{12}{13} - \frac{3}{5}\cdot\frac{5}{13} = \frac{33}{65}.$

D. Suppose that $\Theta \in [90^\circ; 180^\circ]$ and $\Phi \in [270^\circ; 360^\circ]$.

$$\cos\theta = -\sqrt{1-\sin^2\theta} = -\sqrt{1-\left(\frac{4}{5}\right)^2} = -\frac{3}{5}$$

$$\sin^2\Phi + \cos^2\Phi = 1$$

$$\sin\Phi = -\sqrt{1-\cos^2\Phi} = -\sqrt{1-\left(\frac{12}{13}\right)^2} = -\frac{5}{13}$$

Sine of sum:

$\sin(\theta+\Phi) = \sin\theta\cos\Phi + \cos\theta\sin\Phi = \frac{4}{5}\cdot\frac{12}{13} + \frac{3}{5}\cdot\frac{5}{13} = \frac{63}{65}$.

Answer:

$\sin(\theta+\Phi) = \frac{63}{65}$, when $\Theta \in [0^\circ; 90^\circ]$, $\Phi \in [0^\circ; 90^\circ]$ or $\Theta \in [90^\circ; 180^\circ]$, $\Phi \in [270^\circ; 360^\circ]$;

$\sin(\theta+\Phi) = \frac{33}{65}$, when $\Theta \in [0^\circ; 90^\circ]$, $\Phi \in [270^\circ; 360^\circ]$ or $\Theta \in [90^\circ; 180^\circ]$, $\Phi \in [0^\circ; 90^\circ]$.