Answer on Question #48001 – Math – Algebra:

I'm thinking of five distinct integers: a<b<c<d<e. When I sum all possible pairs of these numbers, I get the values 2,3,6,7,9,10,11,13,14,17. What are the original numbers?

Solution.

Hence, the least numbers in the list are $a + b$ and $a + c$. The greatest numbers are $c + e$ and $d + e$. So:

$$\begin{cases} a + b = 2 \\ a + c = 3 \\ c + e = 14 \\ d + e = 17 \end{cases}$$

If we sum all numbers from the list we obtain:

$$2 + 3 + 6 + 7 + 9 + 10 + 11 + 13 + 14 + 17 = 92 = (a + b) + (a + c) + (a + d) +$$

$$+(a + e) + (b + c) + (b + d) + (b + e) + (c + d) + (c + e) +$$

$$+(d + e) = 4(a + b + c + d + e);$$

We obtained $4(a + b + c + d + e) = 92$, divide both sides by 4 and obtain

$$a + b + c + d + e = 23 \tag{1}$$

Use equations $a + b = 2$, $d + e = 17$ and plug their left-hand sides into equation (1).

So:

$$a + b + c + d + e = 23 \Rightarrow 2 + c + 17 = 23 \Rightarrow c = 4;$$

Plug $c = 4$ in equations $a + c = 3$ and $c + e = 14$:

$$\begin{cases} c = 4 \\ a + c = 3 \\ c + e = 14 \end{cases} \Rightarrow \begin{cases} a = -1 \\ e = 10 \end{cases};$$

Plug $a = -1$ in equation $a + b = 2$:

$$\begin{cases} a = -1 \\ a + b = 2 \end{cases} \Rightarrow b = 3;$$

Plug $e = 10$ in equation $d + e = 2$:

$$\begin{cases} e = 10 \\ d + e = 17 \end{cases} \Rightarrow d = 7.$$

Answer.

$a = -1, b = 3, c = 4, d = 7, e = 10.$