Answer on Question #47762 – Math – Algebra

Derive the quadratic formula. Get the x.

Solution

We start with the original quadratic function of the form:

$$y = ax^2 + bx + c$$

In our problem we need to derive the quadratic formula, this mean we have to solve by finding its roots, or the points at which the graph of the parabola hits the x-axis known as the x-intercepts. So, to find the roots or x-intercepts of the equation $y = ax^2 + bx + c$ we need to put y=0. Thus we obtained the following equation.

$$ax^2 + bx + c = 0$$

Then we subtract the constant c from both sides of the equation.

$$ax^2 + bx + c - c = 0 - c$$

$$ax^2 + bx = -c$$

Next step we divide the entire equation by the coefficient of the squared term which is a.

$$\frac{ax^2}{a} + \frac{b}{a}x = \frac{-c}{a}$$

Simplify our equation.

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

We identify the coefficient of the linear term x. Coefficient of the linear term $\frac{b}{a}$. Then we divide it by 2 and raise it to the second power. We square this term.

$$\left(\frac{\frac{b}{a}}{2}\right)^2 = (\frac{b}{2a})^2 = \frac{b^2}{4a^2}$$

We add the obtained result to both sides of the equation.

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$

Simplify the right side of the equation.

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a}\left(\frac{4a}{4a}\right) + \frac{b^2}{4a^2}$$

We open the parenthesis on the right side of the equation.

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2 - 4ac}{4a^2}$$

Then we express the trinomial on the left side of the equation as the square of a binomial.

$$(x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2}$$

We apply square root operations to both sides of the equation to eliminate the exponent 2 of the binomial.

$$\sqrt{(x + \frac{b}{2a})^2} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

We simplify the obtained equation.

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

We subtract term $\frac{b}{2a}$ from both sides of the equation.

$$x + \frac{b}{2a} - \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a} - \frac{b}{2a}$$

Finally we obtained the value of x.

$$x = \pm\frac{\sqrt{b^2 - 4ac}}{2a} - \frac{b}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Thus the value of x is equal to

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Hence, the roots are $x_1 = \frac{-b-\sqrt{b^2-4ac}}{2a}$ and $x_2 = \frac{-b+\sqrt{b^2-4ac}}{2a}$.