

Answer on Question #47726-Math-Statistics and Probability

Customers arrive at a single window server according to a Poisson distribution with mean 10 minutes, and the service time is exponential with mean 6 minutes per customer. Find the following:

- (i) expected number of customers in the system;
- (ii) expected number of customers in the queue;
- (iii) variance of the queue length

Solution

We know that $\lambda = \frac{1}{10}$ per minute and $\mu = \frac{1}{6}$ per minute. Let's work in common units of $minutes^{-1}$.

- (i) expected number of customers in the system is

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{\frac{1}{10}}{\frac{1}{6} - \frac{1}{10}} = \frac{\frac{6}{10}}{1 - \frac{6}{10}} = 1.5.$$

- (ii) expected number of customers in the queue is

$$L_q = \frac{\lambda}{\mu} \frac{\lambda}{\mu - \lambda} = \frac{\frac{1}{10}}{\frac{1}{6}} \cdot 1.5 = 0.9.$$

- (iii) variance of the queue length

$$\sigma^2 = \frac{\frac{\lambda}{\mu}}{\left(1 - \frac{\lambda}{\mu}\right)^2} = \frac{0.6}{(1 - 0.6)^2} = 3.75.$$