Answer on Question #47471 - Math - Statistics and Probability

A chartered Accountant applies for a job in two firms X and Y. He estimates that the probability of his being selected in firm X is 0.7 and being rejected in Y is 0.5 and the probability that at least one of his applications rejected is 0.6. What is the probability that he will be selected in one of the firms.

Solution

Let the event X is "selected at company X" and the event Y is "selected at company Y". This means that X' is "rejected at X" and Y' is "rejected at Y". Here the prime notation means the "complement of". From the information given, we have

$$P(X) = 0.7$$
 and $P(Y') = 0.5$

from which

$$P(X') = 1 - P(X) = 0.3$$
 and $P(Y) = 1 - P(Y') = 0.5$.

As stated, we search for the probability of the event $X \cup Y$ (union of events X and Y - either X or Y, or both, happens (one or the other or both of the applications being selected)). In other words, we need $P(X \cup Y)$, which can be written, using the axioms of probability theory, as

$$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$$

where the \cap symbol means the "intersection of" or, in other words, both X and Y happen. So now we need to calculate $P(X \cap Y)$.

From the information given, we know that the probability of either X' or Y', or both, happening is 0.6, which can be written

$$P(X' \cup Y') = 0.6.$$

From the axioms of set theory we have $(X' \cup Y') = (X \cap Y)'$ - complement of the intersection of X and Y (complement of both X and Y happening). We also have from probability theory that

P(event) = 1 - P(event'), so that

$$P[(X \cap Y)'] = P[(X' \cup Y')] = 1 - P[(X \cap Y)] = 0.6$$

or

$$P[(X \cap Y)] = 1 - 0.6 = 0.4.$$

From the equation above

$$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) = 0.7 + 0.5 - 0.4 = 0.8.$$

Answer: 0.8.