Answer on Question #47278 – Math – Analytic Geometry

PN is any chord of the parabola $y^2 = 4ax$; the point M divides PN in the ratio m:n. find the locus of M.

Solution:

We consider point P on the parabola and N on the x-axis. Let (x_1, y_1) be the point on the parabola, and M will be (x, y). Then we can write the following:

$$y_1^2 = 4ax_1$$

Then we can note the following equity accordingly to the condition of the task:

$$x = x_1$$

$$y = \frac{y_1\, n}{(m + n)}$$

Based on the first formula we can find the square of the y.

$$y^2 = \frac{y_1^2\, n^2}{(m + n)^2}$$

Now we can eliminate the y_1^2 from the first and last equations. We obtained the following result.

$$y^2 = \frac{4ax_1\, n^2}{(m + n)^2}$$

We apply the second equation to eliminate the x_1. We obtained the resulting equation.

$$y^2 = \frac{4ax\, n^2}{(m + n)^2}$$

We can rewrite the noted above equation.

$$y^2 = 4[\frac{an^2}{(m + n)^2}]x$$

$$4ax\, n^2 = y^2(m + n)^2$$

The locus is the parabola $y^2 = 4bx$ where:

$$b = \frac{y^2}{4x} = \frac{4ax\, n^2}{(m + n)^2} \div \frac{4x}{1} = \frac{4ax\, n^2}{(m + n)^2} \cdot \frac{1}{4x}$$

Simplify the equation. Finally we obtained the find value.

$$b = \frac{a\, n^2}{(m + n)^2}$$