Answer on Question #47277 – Math – Analytic Geometry

Show that the straight line $\mathrm{x(p + 2q) + y(p + 3q) = p + q}$ passes through a fixed point for different values of p and q.

Solution:

A line is an infinite geometrical figure. If we extend a line segment at both ends, we get a line. A line is always represented by two arrows at its ends, to indicate its infinite nature.

Mathematically, a line can be represented by a linear equation, that is, an equation of degree one. The most general form of a straight line is

$$ax + by + c = 0;\ a^2 + b^2 \neq 0$$

All the points (x, y) that lie on the line satisfy the equation on that line, and conversely, if a point (x, y) satisfies the equation of a line, it lies on that line.

We can simplify our equation by opening the parenthesis:

$$\mathrm{xp + 2xq + yp + 3yq - p - q = 0}$$

Combine like terms:

$$\mathrm{(xp + yp - p) + (2xq + 3yq - q) = 0}$$

$$\mathrm{p(x + y - 1) + q(2x + 3y - 1) = 0}$$

Thus, the given line passes through the intersection of the lines $\mathrm{x + y - 1 = 0}$ and $\mathrm{2x + 3y - 1 = 0}$.

Now we have to solve the system of the obtained equations.

$$\begin{cases} \mathrm{x + y - 1 = 0} \\ \mathrm{2x + 3y - 1 = 0} \end{cases}$$

Solve system of equations by elimination by addition. We multiply the first equation by -2 and add two equations.

$$\begin{cases} \mathrm{-2x - 2y + 2 = 0} \\ \mathrm{2x + 3y - 1 = 0} \end{cases}$$

$$\mathrm{-2x - 2y + 2 + 2x + 3y - 1 = 0}$$

Simplify by combining like terms:

$\mathrm{-2y + 2 + 3y - 1 = 0}$

$\mathrm{y = -1}$

Now we find the value of x. We can substitute the value of y either in the first or second equation. In our case we choose the first equation.

$$x + (-1) - 1 = 0$$

$$x - 2 = 0$$

$$x = 2$$

The coordinate of the find point will be equal to (2,-1).

We can also check obtained solution. Substitute the values of x and y into the original equations.

$$\begin{cases} 2 + (-1) - 1 = 0 \\ 2(2) + 3(-1) - 1 = 0 \end{cases}$$

Simplify the system of equations.

$$\begin{cases} 2 - 2 = 0 \\ 4 - 3 - 1 = 0 \end{cases}$$

$$\begin{cases} 0 = 0 \\ 0 = 0 \end{cases}$$

Thus we got the true statement. The required coordinate of point are equal (2,-1).

We can also consider different values of p and q. For example, we put p=3 and q=1. Substitute into original equation of the line.

$$x(3 + 2) + y(3 + 3) = 3 + 1$$

Simplify by opening the parenthesis.

$$5x + 6y = 4$$

Now substitute the find coordinate of the x and y.

$$5(2) + 6(-1) = 4$$

$$10 - 6 = 4$$

$$4 = 4$$

We also can consider other values of p and q. For example, p=-2 and q = -1. As in previous part we substitute into original equation. We obtained the following result.

$$x\big(-2 + 2(-1)\big) + y\big(-2 + 3(-1)\big) = -2 - 1$$

Simplify by opening the parenthesis.

$$-4x - 5y = -3$$

Now substitute the find coordinate of the x=2 and y=-1.

$$-4(2) - 5(-1) = -3$$

$$-8 + 5 = -3$$

$$-3 = -3$$

We again got the true statement.

In this way our system has a unique solution therefore for any values of p and q line $x(p + 2q) + y(p + 3q) = p + q$ passes through a fixed point with coordinates (2,-1).