Answer on Question #47137 – Math – Calculus

Question:

If f(x) equal to (1/x) to the power x show that f''(1) equal to 0.

Solution:

$$f(x) = \left(\frac{1}{x}\right)^x = \frac{1}{x^x} = x^{-x}$$

$f(x) = (u(x))^{v(x)}$, then $f'(x) = (u(x))^{v(x)}\left(v'(x)\ln u(x) + \frac{u'(x)v(x)}{u(x)}\right)$

In our case, $u(x) = x,\ v(x) = -x$, then $f'(x) = (x^{-x})' = -x^{-x}\cdot(\ln x + 1)$

And the second derivative will be: $f''(x) = -\left(x^{-x}\cdot(\ln x + 1)\right)' = x^{-x-1}\left(1 - x(\ln(x)+1)^2\right)$

After that we can calculate $f''(1)$. So:

$$f''(1) = 1^{-1-1}\left(1 - 1(\ln(1)+1)^2\right) = 1^{-2}\left(1 - 1(0+1)^2\right) = 0$$