Answer on Question #47129 - Math – Calculus

Show that $\frac{x^2}{2}\log(1+x) < \frac{x^2}{2}(1+x)$ for any $x > 0$.

$$\frac{x^2}{2}\log(1+x) < \frac{x^2}{2}(1+x)$$

$$\frac{x^2}{2}\big(\log(1+x) - (1+x)\big) < 0 \quad (1)$$

$\frac{x^2}{2} > 0$ for any $x > 0$, hence we can rewrite inequality:

$$f(x) = \log(1+x) - (1+x) < 0 \quad (2)$$

Particular case of $x = 0$: $\log(1+0) - (1+0) = \log(1) - 1 = -1 < 0$.
(Note that inequality (1) turns into 0)

As soon as we know that (2) true for $x = 0$, the only thing we need to show is that $f(x)$ decreases as x increases. Mathematically: $\frac{df(x)}{dx} < 0$ for any $x > 0$.

$$\frac{df(x)}{dx} = \frac{d(\log(1+x))}{dx} - \frac{d(1+x)}{dx} = \frac{1}{1+x} - 1 < 0$$

$$\frac{1}{1+x} < 1$$

For any $x > 0$: $1 + x > 0$, hence we can write:

$$1 < 1 + x$$

$$0 < x$$

Thus, $\frac{df(x)}{dx} < 0$ and $\frac{x^2}{2}\log(1+x) < \frac{x^2}{2}(1+x)$ for any $x > 0$.