

## Answer on Question #47128 – Math – Calculus

### Question.

Find the length of the curve  $y = \log \sec x$  between the points  $x = 0$  and  $x = \frac{\pi}{3}$ .

### Solution.

We shall use the next formula:  $L = \int_0^{\frac{\pi}{3}} \sqrt{1 + (y'(x))^2} dx$ .

$$y'(x) = \left( \log \frac{1}{\cos x} \right)' = \cos x \cdot \left( -\frac{-\sin x}{\cos^2 x} \right) = \frac{\sin x}{\cos x}.$$

$$L = \int_0^{\frac{\pi}{3}} \sqrt{1 + \frac{\sin^2 x}{\cos^2 x}} dx = \int_0^{\frac{\pi}{3}} \sqrt{\frac{1}{\cos^2 x}} dx = \int_0^{\frac{\pi}{3}} \frac{dx}{\cos x} = \int_0^{\frac{\pi}{3}} \frac{\cos x}{\cos^2 x} dx = \int_0^{\frac{\pi}{3}} \frac{d(\sin x)}{1 - \sin^2 x} =$$

$$= -\frac{1}{2} \log \left| \frac{\sin x - 1}{\sin x + 1} \right| \Big|_0^{\frac{\pi}{3}} = -\frac{1}{2} \log \left| \frac{\frac{\sqrt{3}}{2} - 1}{\frac{\sqrt{3}}{2} + 1} \right| = \log(\sqrt{3} + 2).$$

**Answer.**  $\log(\sqrt{3} + 2)$