Answer on Question #47124 – Mathematics – Calculus

Question:

Find the area of the region enclosed by the curves $y_1(x) = x^2$ and $y_2(x) = \frac{1}{2}(x^2 + x)$.

Solution:

Let graph the given curves and find their interception points.

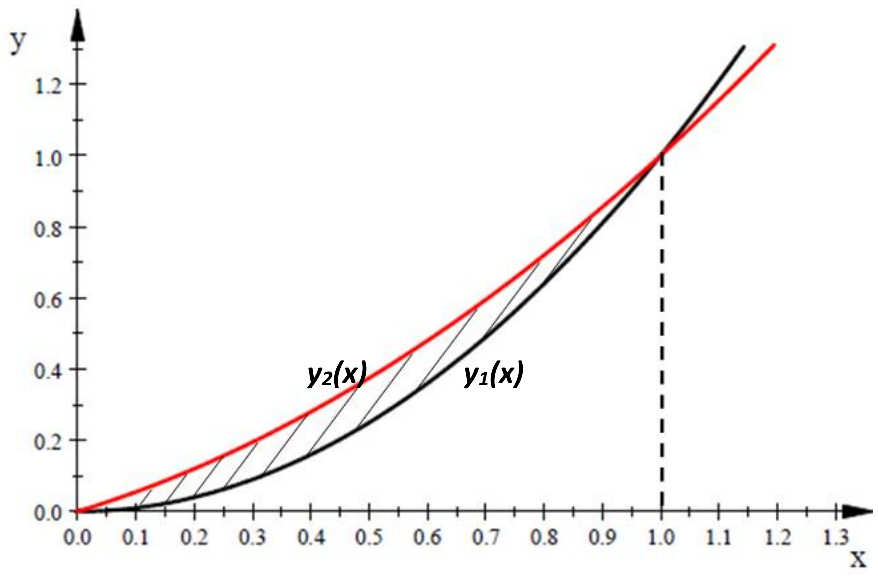

Fig.1

$$x^2 = \frac{1}{2}(x^2 + x) \Longrightarrow x^2 - \frac{1}{2}x^2 - \frac{1}{2}x = 0 \Longrightarrow \frac{1}{2}x^2 - \frac{1}{2}x = 0 \Longrightarrow x(x-1) = 0 \Longrightarrow \ x_1 = 0, \qquad x_2 = 1.$$

Therefore, the interception points are $x_1 = 0$, $x_2 = 1$.

By definition, the area of the region between curves *y=f(x)* and *y=g(x)* on the interval [*a, b*] (assuming that $f(x) \geq g(x)$) is defined by the following formula

$$S = \int_a^b (f(x) - g(x))dx.$$

Since $y_2(x) = \frac{1}{2}(x^2 + x) > y_1(x) = x^2$ (see fig.1), then we have

$$S = \int_0^1 (\frac{1}{2}(x^2 + x) - x^2)dx = \int_0^1 (\frac{1}{2}x - \frac{1}{2}x^2)dx = \left(\frac{x^2}{4} - \frac{x^3}{6}\right)|_0^1 = \frac{1}{4} - \frac{1}{6} = \frac{2}{24} = \frac{1}{12}$$
$$\cong 0.08 \text{ square units}$$

Answer: $S = \frac{1}{12} \cong 0.08$ square units.