Answer on Question #47091 – Math – Statistics and Probability

Question.

True or False. Give reason.

The mean and variance of then Poison distribution are equal.

Solution.

Let ξ be a random variable which has the distribution of Poisson with rate $\lambda > 0$. Then $P(\xi = k) = \frac{\lambda^k}{k!} e^{-\lambda}, k = 0, 1, 2, \ldots$ Then we have:

The mean is $E\xi = \sum_{k=0}^{\infty} k \frac{\lambda^k}{k!} e^{-\lambda} = e^{-\lambda} \sum_{k=1}^{\infty} \frac{\lambda^k}{(k-1)!} = \lambda e^{-\lambda} \sum_{k=1}^{\infty} \frac{\lambda^{k-1}}{(k-1)!} = \lambda e^{-\lambda} e^{\lambda} = \lambda.$

Calculate

$$E\xi^2 = \sum_{k=0}^{\infty} k^2 \frac{\lambda^k}{k!} e^{-\lambda} = e^{-\lambda} \sum_{k=1}^{\infty} k \frac{\lambda^k}{(k-1)!} = e^{-\lambda} \sum_{k=1}^{\infty} (k - 1 + 1) \frac{\lambda^k}{(k-1)!} =$$

$$= e^{-\lambda} \sum_{k=1}^{\infty} (k-1) \frac{\lambda^k}{(k-1)!} + e^{-\lambda} \sum_{k=1}^{\infty} \frac{\lambda^k}{(k-1)!} = \lambda^2 e^{-\lambda} \sum_{k=2}^{\infty} \frac{\lambda^{k-2}}{(k-2)!} + E\xi = \lambda^2 e^{-\lambda} e^{\lambda} + \lambda = \lambda^2 + \lambda$$

The variance is

$Var\xi = E\xi^2 - (E\xi)^2 = \lambda^2 + \lambda - \lambda^2 = \lambda$. We see that $E\xi = Var\xi = \lambda.$

Answer. True.