Answer on Question #46774 – Math - Linear Algebra

Solve the set of linear equations by the matrix method: a+3b+2c=3, 2a-b-3c= -8, 5a+2b+c=9. Solve for c.

Solution

$$\begin{cases} \mathrm{a} + 3\mathrm{b} + 2\mathrm{c} = 3 \\ 2\mathrm{a} + (-1)\mathrm{b} + (-3)\mathrm{c} = -8 \\ 5\mathrm{a} + 2\mathrm{b} + \mathrm{c} = 9 \end{cases}$$

First let

$$A = \begin{pmatrix} 1 & 3 & 2 \\ 2 & -1 & -3 \\ 5 & 2 & 1 \end{pmatrix}.$$

This is the matrix formed by the coefficients of the given system of equations. The matrix method gives $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = A^{-1} \begin{pmatrix} 3 \\ -8 \\ 9 \end{pmatrix}$.

Take note that the right hand values of the system are 3, -8, and 9 and they are highlighted here:

$$\begin{cases} \mathrm{a} + 3\mathrm{b} + 2\mathrm{c} = \mathbf{3} \\ 2\mathrm{a} + (-1)\mathrm{b} + (-3)\mathrm{c} = -\mathbf{8} \\ 5\mathrm{a} + 2\mathrm{b} + \mathrm{c} = \mathbf{9} \end{cases}$$

These values are important as they will be used to replace the columns of the matrix A.

Now let's calculate the determinant of the matrix A

$$\det A = \begin{vmatrix} 1 & 3 & 2 \\ 2 & -1 & -3 \\ 5 & 2 & 1 \end{vmatrix} = 1 \cdot 1 \cdot (-1) + 3 \cdot 5 \cdot (-3) + 2 \cdot 2 \cdot 2 - 2 \cdot 5 \cdot (-1) - 2 \cdot 1 \cdot 3 - 1 \cdot 2 \cdot (-3)$$
$$= -28.$$

Now replace the third column of A (that corresponds to the variable 'c') with the values that form the right hand side of the system of equations. We will denote this new matrix A_c (since we're replacing the 'c' column so to speak).

Now compute the determinant of A_c

$$\det A_c = \begin{vmatrix} 1 & 3 & 3 \\ 2 & -1 & -8 \\ 5 & 2 & 9 \end{vmatrix} = 1 \cdot 9 \cdot (-1) + 3 \cdot 5 \cdot (-8) + 2 \cdot 2 \cdot 3 - 3 \cdot 5 \cdot (-1) - 2 \cdot 9 \cdot 3 - 1 \cdot 2 \cdot (-8)$$
$$= -140.$$

To find the solution for c, divide the determinant of A_c by the determinant of A to get:

$$c = \frac{\det A_c}{\det A} = \frac{-140}{-28} = 5.$$

Answer: 5.