Answer on Question #46771 – Mathematics – Trigonometry

Question:

Prove the trigonometric identity

$$\frac{4tanA}{1 - tan^4A} = tan2A + sin2A$$

Solution:

Let`s prove the given trigonometric identity in several steps.

1) Rewrite the denominator in a left side of identity as a product of two factors:

$$\frac{4tanA}{1-tan^4A} = \frac{4tanA}{(1-tan^2A)(1+tan^2A)} \qquad (1)$$

2) Use the Pythagorean identity $1 + tan^2A = \frac{1}{cos^2A}$ to simplify the expression in the denominator of (1):

$$\frac{4tanA}{(1-tan^2A)(1+tan^2A)} = \frac{4tanA \cdot cos^2A}{1-tan^2A} \qquad (2)$$

3) Use the tangent double-angle formula $tan2A = \frac{2tanA}{1-tan^2A}$ to rewrite the right side of (2):

$$\frac{4tanA \cdot cos^2A}{1-tan^2A} = tan2A \cdot 2cos^2A. \qquad (3)$$

4) Using the cosine power-reduction formula $cos^2A = \frac{1+cos2A}{2}$ and representing the tangent of an angle as the ratio of the sine to the cosine: $tanA = \frac{sinA}{cosA}$, we obtain

$$tan2A \cdot 2cos^2A = tan2A \cdot 2\left(\frac{1 + cos2A}{2}\right) = tan2A \cdot (1 + cos2A) = tan2A + tan2A \cdot cos2A$$
$$= tan2A + \frac{sin2A}{cos2A} \cdot cos2A = tan2A + sin2A.$$

Thus, we prove that

$$\frac{4tanA}{1 - tan^4A} = \frac{4tanA}{(1 - tan^2A)(1 + tan^2A)} = \frac{4tanA \cdot cos^2A}{1 - tan^2A} = tan2A \cdot 2cos^2A = tan2A \cdot 2\left(\frac{1 + cos2A}{2}\right)$$
$$= tan2A \cdot (1 + cos2A) = tan2A + tan2A \cdot cos2A = tan2A + \frac{sin2A}{cos2A} \cdot cos2A$$
$$= tan2A + sin2A,$$

i.e.

$$\frac{4tanA}{1 - tan^4A} = tan2A + sin2A$$

Answer: The given trigonometric identity is proved.