Answer on Question #46738 – Math – Linear Algebra

Problem.

1. Show that: [x l m 1]
   [a x n 1] = (x-a)(x-b)(x-c)
   [a b x 1]
   [a b c 1]

2. Show that: [1+ (a)^2 - (b)^2 2b -2b]
   [2ab 1 - (a)^2 + (b)^2 2a]
   [2b -2(a)^2 1 - (a)^2 - (b)^2]
   is a perfect cube.

   {please note that these are determinants, the first one is a 4X4 determinant and the second one is a 3X3 determinant}

Solution:

1.

$$\begin{bmatrix} x & l & m & 1 \\ a & x & n & 1 \\ a & b & x & 1 \\ a & b & c & 1 \end{bmatrix} \sim \begin{bmatrix} x-a & l-b & m-c & 0 \\ 0 & x-b & n-c & 0 \\ 0 & 0 & x-c & 0 \\ a & b & c & 1 \end{bmatrix}$$

Hence

$$\det \begin{bmatrix} x-a & l-b & m-c & 0 \\ 0 & x-b & n-c & 0 \\ 0 & 0 & x-c & 0 \\ a & b & c & 1 \end{bmatrix} = 1 \cdot \det \begin{bmatrix} x-a & l-b & m-c \\ 0 & x-b & n-c \\ 0 & 0 & x-c \end{bmatrix} = (x-a)(x-b)(x-c)$$

Then

$$\det \begin{bmatrix} x & l & m & 1 \\ a & x & n & 1 \\ a & b & x & 1 \\ a & b & c & 1 \end{bmatrix} = (x-a)(x-b)(x-c)$$

2.

$$\det \begin{bmatrix} 1+a^2-b^2 & 2b & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a^2 & 1-a^2-b^2 \end{bmatrix}$$

isn't a perfect cube for all a and b, as for $a = 2$ and $b = 0$

$$\det \begin{bmatrix} 1+a^2-b^2 & 2b & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a^2 & 1-a^2-b^2 \end{bmatrix}$$

$$= \det \begin{bmatrix} 5 & 0 & 0 \\ 0 & -3 & 4 \\ 0 & -8 & -3 \end{bmatrix} = 205$$

and 205 isn't a perfect cube.