Answer on Question #46679 – Math – Linear Algebra

a) Find the inverse of the matrix

1 2 2

1 2 4

1 1 1

A

using Gauss-Jordon method.

Solution.

To calculate inverse matrix, we need to do the following steps.

- **Set the matrix and append the identity matrix of the same dimension to it.**
- **Reduce the left matrix to row echelon form using elementary row operations for the whole matrix (including the right one).**

As a result, we will get the inverse calculated on the right.

If a determinant of the main matrix is zero, inverse doesn't exist.

Thus:

$$\begin{pmatrix} 1 & 2 & 2 \\ 1 & 2 & 4 \\ 1 & 1 & 1 \end{pmatrix} \to \left(\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 1 & 2 & 4 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 1 & 2 & 4 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \to$$ **(Subtract the 1st row from the 2nd and 3rd)** $$\to \left(\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & -1 & 1 & 0 \\ 0 & -1 & -1 & -1 & 0 & 1 \end{array}\right) \to$$

(Find the pivot in the 2nd column (inversing the sign in the whole row) and swap the 3rd and the 2nd rows) $$\to \left(\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 2 & -1 & 1 & 0 \end{array}\right) \to$$ **(Multiply the 2nd row by 2 and subtract the 2nd row from the 1st row)** $\to$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & -2 \\ 0 & 2 & 2 & 2 & 0 & 2 \\ 0 & 0 & 2 & -1 & 1 & 0 \end{array}\right) \rightarrow$$ **(Subtract the 3rd row from the 2nd row and divide 2nd and 3rd rows by 2)**

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & \frac{3}{2} & -1/2 & 1 \\ 0 & 0 & 1 & -\frac{1}{2} & \frac{1}{2} & 0 \end{array}\right)$$

So, $$\begin{pmatrix} 1 & 2 & 2 \\ 1 & 2 & 4 \\ 1 & 1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} -1 & 0 & -2 \\ \frac{3}{2} & -\frac{1}{2} & 1 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$$