Answer on Question #46505 – Math – Statistics and Probability

Ten individuals are chosen at random, from a normal population and their (5) weights (in kg) are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71 and 71. In the light of this data set, test the claim that the mean weight in population is 66 kg at 5% level of significance.

[The following tά values are given]

degree of freedom ά = 0.05 ά = 0.025

9	1.83	2.26
10	1.81	2.23

Solution

Mean weight is

$$\bar{x} = \frac{63+63+66+67+68+69+70+70+71+71}{10} = 67.8\,kg$$

To get $\bar{x}$, put =AVERAGE(63;63;66;67;68;69;70;70;71;71) in Excel, output is 67.8

Standard deviation is

$$s = \sqrt{\frac{\sum x^2 - n\bar{x}^2}{n-1}}.$$

$$\sum x^2 = 63^2+63^2+66^2+67^2+68^2+69^2+70^2+70^2+71^2+71^2 = 46050.$$

$$s = \sqrt{\frac{46050 - 10\cdot 67.8^2}{10-1}} = 3.0\,kg.$$

To get s, put = STDEV(63;63;66;67;68;69;70;70;71;71) in Excel, the output is 3,011091.

Hypotheses

$$H_0:\ \mu = 66$$

$$H_1: \mu \neq 66$$

Test statistic

$$t = \frac{\bar{x} - 66}{\frac{s}{\sqrt{n}}} = \frac{67.8 - 66}{\frac{3.0}{\sqrt{10}}} = 1.90.$$

To get t, put =(67,8-66)/(3/SQRT(10)) in Excel, the output is 1.897367.

Method 1

Critical value approach

Reject H_0 if $|t| \geq t_{\frac{\alpha}{2},n-1}$.

We have $10 - 1 = 9$ degrees of freedom and the two-tailed test.

Using statistical tables

$$t_{critical} = t_{0.025,9} = 2.26.$$

To get $t_{critical}$, put =TINV(0,05;9) in Excel, the output is 2.622157.

Since $t < t_{critical}$, thus we fail to reject the null hypothesis, hence the mean weight in population is 66 kg at 5% level of significance.

Method 2

p-value approach

Reject H_0 if p-value $\leq \alpha$,

in this problem $\alpha = 0.05$.

Generally speaking

In two tail hypotheses p-value=2*$P(T \geq |t|)$

To get p-value(two tail) put =TDIST(t,n-1,2) in Excel.

In this problem to get p-value(two tail) put =TDIST(1,9;9;2) in Excel, the output is 0.089.

Because 0.089>0.05, then we fail to reject H_0, hence the mean weight in population is 66 kg at 5% level of significance.

Method 3

Confidence intervals approach

Reject H_0 If μ_0 =66 (the number you were checking) is not in the confidence interval.

95 % Confidence interval for sample mean is $\left(\bar{x} - t_{\frac{\alpha}{2};n-1}\frac{s}{\sqrt{n}};\ \bar{x} + t_{\frac{\alpha}{2};n-1}\frac{s}{\sqrt{n}}\right)$.

To get $\bar{x} - t_{\frac{\alpha}{2};n-1}\frac{s}{\sqrt{n}}$ put = 67,8-TINV(0,05;9)*3/SQRT(10) in Excel, output is 65.65393.

To get $\bar{x} - t_{\frac{\alpha}{2};n-1}\frac{s}{\sqrt{n}}$ put = 67,8+TINV(0,05;9)*3/SQRT(10) in Excel, output is 69.94607.

Because 66 is in confidence interval $(65.65393; 69.94607)$, therefore we fail to reject H_0, hence the mean weight in population is 66 kg at 5% level of significance.