Question. Find the direction in which the function $f = x^2 - y^2 - 2xy$ decreases most rapidly at the point $(1, 1)$.

Solution. Recall that the direction in which the function f *increases* most rapidly at the point $(1, 1)$ is given by the gradient,

$$\nabla f = (f'_x, f'_y),$$

of f at that point.

Therefore the direction in which the function f *decreases* most rapidly at the point $(1, 1)$ is *opposite to the gradient*, i.e.

$$-\nabla f = (-f'_x, -f'_y).$$

We have that

$$f'_x = 2x - 2y, \qquad f'_y = -2y - 2x,$$

and so

$$f'_x(1, 1) = 2 \cdot 1 - 2 \cdot 1 = 2 - 2 = 0,$$
$$f'_y(1, 1) = -2 \cdot 1 - 2 \cdot 1 = -2 - 2 = -4.$$

Hence the direction in which the function f decreases most rapidly at the point $(1, 1)$ is

$$-\nabla f = -(0, -4) = (0, 4).$$

Answer. $(0, 4)$.