One half percent of the population has a particular disease. A test is developed for the disease. The test gives a false positive 3% of the time and a false negative 2% of the time.

**(a).** What is the probability that Joe (a random person) tests positive?

**(b).** Joe just got the bad news that the test came back positive; what is the probability that Joe has the disease?

**Solution**

Let $D$ be the event that Joe has the disease. Let $T$ be the event that Joe's test comes back positive. We are told that $P(D) = 0.005$, since $\frac{1}{2}$% of the population has the disease, and Joe is just an average guy. We are also told that $P(T|D) = 0.98$, since 2% of the time a person having the disease is missed ("false negative"). We are told that $P(T|D^c) = 0.03$, since there are 3% false positives.

**(a).** We want to compute $P(T)$. We do so by conditioning on whether or not Joe has the disease:

$$P(T) = P(T|D)P(D) + P(T|D^c)P(D^c) = (0.98)(0.005) + (0.03)(0.995) = 0.035.$$

**(b).** We want to compute

$$P(D|T) = \frac{P(D \cap T)}{P(T)} = \frac{P(T|D)P(D)}{P(T|D)P(D) + P(T|D^c)P(D^c)} = \frac{(0.98)(0.005)}{(0.98)(0.005) + (0.03)(0.995)} = 0.14.$$