

Answer on Question # 42970, Math, Discrete Mathematics

Task:

Expand the following Boolean functions into their canonical form:

i. $f(X,Y,Z) = XY + YZ + X'Z + X'Y$

ii. $f(X,Y,Z) = XY + X'Y + X'YZ$

Solution:

i. $f(X,Y,Z) = XY + YZ + X'Z + X'Y$

A table about the minterm:

Input variables			Minterm	
X	Y	Z	Product term	
0	0	0	$X'Y'Z'$	m_0
0	0	1	$X'Y'Z$	m_1
0	1	0	$X'YZ'$	m_2
0	1	1	$X'YZ$	m_3
1	0	0	$XY'Z'$	m_4
1	0	1	$XY'Z$	m_5
1	1	0	XYZ'	m_6
1	1	1	XYZ	m_7

Write the truth table:

X	Y	Z	X'	XY	YZ	X'Z	X'Y	f
0	0	0	1	0	0	0	0	0
0	0	1	1	0	0	1	0	1
0	1	0	1	0	0	0	1	1
0	1	1	1	0	1	1	1	1
1	0	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0
1	1	0	0	1	0	0	0	1
1	1	1	0	1	1	0	0	1

To expand the following Boolean function into its canonical form we have to sum all the minterms (where the f is equal to 1):

$$\text{So, } f(X,Y,Z) = X'Y'Z + X'YZ' + X'YZ + XYZ' + XYZ$$

ii. $f(X,Y,Z) = XY + X'Y + X'YZ$

Analogically:

X	Y	Z	X'	XY	X'Y	X'YZ	f
0	0	0	1	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	1	0	1
0	1	1	1	0	1	1	1
1	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0
1	1	0	0	1	0	0	1
1	1	1	0	1	0	0	1

$$\text{So, } f(X,Y,Z) = X'YZ' + X'YZ + XYZ' + XYZ$$

Answer:

i. $f(X,Y,Z) = X'Y'Z + X'YZ' + X'YZ + XYZ' + XYZ$

ii. $f(X,Y,Z) = X'YZ' + X'YZ + XYZ' + XYZ$

