Answer on Question#38388 – Math - Calculus

A vector has direction angles α = 85° and β = 65°

a) Find the value of γ?

b) Find a vector that has those direction angles?

c) Explain why it is not possible for two of a vector's direction angles to be less than 45°?

Solution

a) From the equation $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ we can find γ:

$$\cos\gamma = \pm\sqrt{1-\cos^2\alpha-\cos^2\beta} = \pm\sqrt{1-\cos^2 85°-\cos^2 65°}$$

$$\cos\gamma = \pm 0.902 \to \gamma = \cos^{-1} 0.902 = 25.6° \; or \; \gamma = \cos^{-1}(-0.902) = 154.4°.$$

b) We can find a vector that has those direction angles from the equation

$$\frac{\cos\alpha}{V_x} = \frac{\cos\beta}{V_y} = \frac{\cos\gamma}{V_z} = \frac{1}{|\vec{V}|}.$$

A vector that has those direction angles and $|\vec{V}| = V$ – any positive constant $\vec{V}$ is

$$\vec{V} = (V\cos 85°, V\cos 65°, V\cos 25.6°) or$$

$$\vec{V} = (V\cos 85°, V\cos 65°, V\cos 154.4°).$$

So

$$\vec{V} = (V\cdot 0.087, V\cdot 0.423, V\cdot 0.902) or$$

$$\vec{V} = (V\cdot 0.087, V\cdot 0.423, -V\cdot 0.902).$$

c) If two of a vector's direction angles are less than 45° their cosines will be greater then $\cos 45° = \frac{1}{\sqrt{2}}$.

From the equation $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$:

$$\cos^2\gamma = 1-\cos^2\alpha-\cos^2\beta < 1-\frac{1}{2}-\frac{1}{2} = 0.$$

In that case $\cos^2\gamma < 0$. That's why it doesn't right.