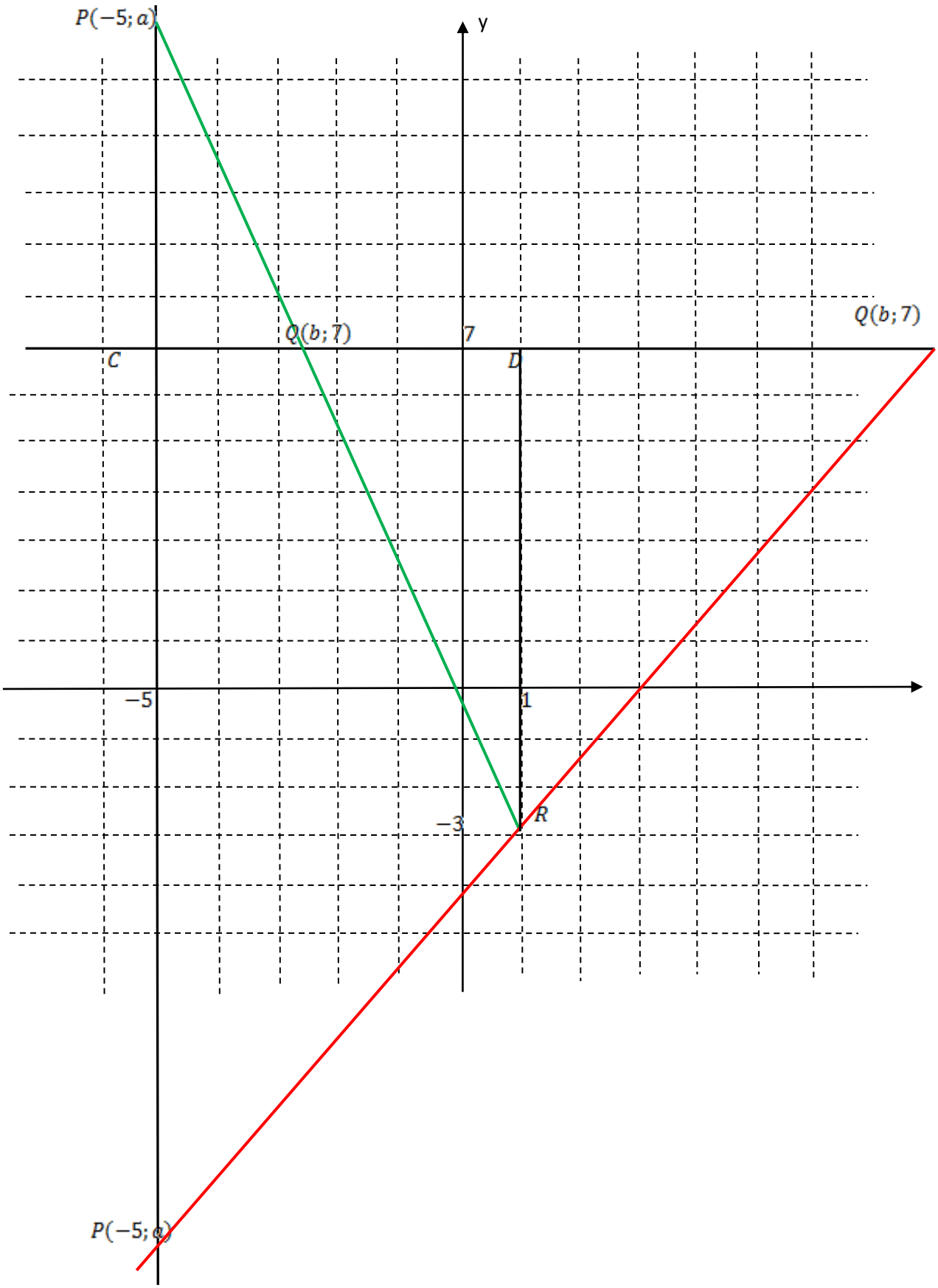
P(-5,a),Q(b,7),R(1,-3)points are collinear such that PQ=QR. Find the value of a & b.

Solution:



There are two cases: green line and red line. The second case (red line) is impossible because $PQ \neq QR$. Thus we have only one case (green line). Further

$$\angle PQC = \angle RQD = \alpha$$

because these angles are vertical ones. And we have

$$\angle CPQ = \pi - \angle PCQ - \angle PQC = \pi - \frac{\pi}{2} - \alpha = \frac{\pi}{2} - \alpha;$$

$$\angle DRQ = \pi - \angle RDQ - \angle PQD = \pi - \frac{\pi}{2} - \alpha = \frac{\pi}{2} - \alpha.$$

Thus $\angle CPQ \cong \angle DRQ$. If $PQ = QR$ then $\Delta RDQ \cong \Delta PCQ$ (by Angle-Side-Angle (ASA) Congruence). So

$$QC = DQ,$$

$$b - (-5) = 1 - b,$$

$$b + 5 = 1 - b,$$

$$2b = -4,$$

$$\boxed{b = -2}$$

Also we have

$$DR = CP,$$

$$7 - (-3) = a - 7,$$

$$7 + 3 = a - 7,$$

$$\boxed{a = 17}$$

Answer:

$$a = 17, \qquad b = -2.$$