Answer on Question #34550 - Math - Differential Equations

Find the solution of the equation that satisfies the given conditions for x-> infinity. The equation is $(x^2)(y') - \cos(2y) = 1$, y(+infinity) = (9)(pie)/(4)

Solution

$$\begin{split} x^2y'-cos(2y)&=1\ \rightarrow\ \frac{dy}{1+cos(2y)}=\frac{dx}{x^2}\ \rightarrow\\ &\rightarrow\ \int\frac{dy}{1+cos(2y)}=\int\frac{dx}{x^2}\ \rightarrow\ \frac{tan(y)}{2}=-\frac{1}{x}+c\ \rightarrow\\ &\rightarrow tan(y)=-\frac{2}{x}+2c\\ &\rightarrow\ y=tan^{-1}\left(\frac{2cx-2}{x}\right)\\ &\text{If }y(\infty)=tan^{-1}(2c)=\frac{9\pi}{4}\text{, then}\\ &tan\big(y(\infty)\big)=tan\left(tan^{-1}(2c)\right)=2c=tan\left(\frac{9\pi}{4}\right)=1 \end{split}$$

$$\rightarrow 2c = 1 \rightarrow c = \frac{1}{2}$$

Thus,
$$y = tan^{-1} \left(\frac{x-2}{x}\right)$$
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