1. $\frac{\sin A}{1-\cos A}+\frac{\tan A}{1+\cos A}=2\sec A\cos ecA+\cot A$. Prove the identity.

Solution.

The left-hand part of the identity:

$$\frac{\sin A}{1-\cos A}+\frac{\tan A}{1+\cos A}=\frac{\sin A(1+\cos A)+\tan A(1-\cos A)}{(1-\cos A)(1+\cos A)}=\frac{\sin A+\sin A\cos A+\tan A-\sin A}{1-\cos^2 A}=$$

$$=\frac{\sin A\cos A+\tan A}{\sin^2 A}=\frac{\sin A\left(\cos A+\frac{1}{\cos A}\right)}{\sin^2 A}=\frac{\frac{\cos^2 A+1}{\cos A}}{\sin A}=\frac{\cos^2 A+1}{\sin A\cos A}.$$

The right-hand part of the identity:

$$2\sec A\cos ecA+\cot A=\frac{2}{\cos A\sin A}+\frac{\cos A}{\sin A}=\frac{2+\cos^2 A}{\sin A\cos A}.$$

Answer: the identity is wrong.