$$f(x) = x^2 + bx + c$$

$$N = \max|f(x)|, x \in [-10,10]$$

The maximum $|f(x)|$ is reached at boundary points $-10,10$ or at inner local extrema point.

Let's consider N as function of b, c: $N(b, c)$. Since the interval $[-10; 10]$ is symmetric about 0 and $x^2 + bx + c = x(x + b) + c$ N is even function of b:

$N(b, c) = N(-b, c)$. Since N reaches minimum the minima point of b is $b = 0$. So $f(x) = x^2 + c$.

$$f(-10) = f(10) = 100 + c; f(0) = c$$

Thus

$$N = \min_{c} \max_{x} |f(x, c)|$$

Minimum of N is reached if

$$|100 + c| = |c|$$

So

$$c = -50$$

$$N = 50$$