

Task. Assuming that Lxy is a means “x loves y” and Rx “x is rich” write logical formulas for

- a) it is enough to love someone to be rich,
- b) being rich doesn't mean loving someone,
- c) you are not rich if you love no one,
- d) love is not always a symmetrical relation.

Solution.

a) The phrase

it is enough to love someone to be rich

can be reformulated as follows:

if exists y such that x loves y, then x is rich

Using logical formulas this can be written in the following way:

$$\forall x : \left[(\exists y : Lxy) \Rightarrow Rx \right].$$

b) The phrase

being rich doesn't mean loving someone

can not be formulated as follows:

there exists x which is rich and which loves no one.

In terms of logical formulas this means that

$$\exists x : (Rx \wedge (\forall y : \overline{Lxy})).$$

c) The phrase

you are not rich if you love no one

can not be formulated as follows:

if x loves no one, then x is not rich.

In terms of logical formulas this means that

$$\forall x : (\forall y : \overline{Lxy}) \Rightarrow \overline{Rx},$$

$$\forall x : (\forall y : \overline{Lxy}) \vee \overline{Rx},$$

$$\forall x : (\exists y : Lxy) \vee \overline{Rx}.$$

d) The phrase

love is not always a symmetrical relation

can be reformulated as follows:

there exist x and y such that (x loves y) and (y does not love x).

Using logical formulas we obtain the following expression:

$$\exists x \exists y : (Lxy \wedge \overline{Lyx}).$$