

$$\frac{\sin a - \sin b}{\cos a + \cos b} + \frac{\cos a - \cos b}{\sin a + \sin b} = \frac{(\sin a - \sin b)(\sin a + \sin b) + (\cos a - \cos b)(\cos a + \cos b)}{(\cos a + \cos b)(\sin a + \sin b)} \quad (1)$$

We know that $(a - b)(a + b) = a^2 - b^2$ (2)

So, using formula (2):

$$(\sin a - \sin b)(\sin a + \sin b) = \sin^2 a - \sin^2 b \quad (3)$$

$$(\cos a - \cos b)(\cos a + \cos b) = \cos^2 a - \cos^2 b \quad (4)$$

(3),(4) \rightarrow (1)

$$\begin{aligned} \frac{(\sin a - \sin b)(\sin a + \sin b) + (\cos a - \cos b)(\cos a + \cos b)}{(\cos a + \cos b)(\sin a + \sin b)} &= \frac{\sin^2 a - \sin^2 b + \cos^2 a - \cos^2 b}{(\cos a + \cos b)(\sin a + \sin b)} = \\ &= \frac{\sin^2 a + \cos^2 a - \sin^2 b - \cos^2 b}{(\cos a + \cos b)(\sin a + \sin b)} = \frac{(\sin^2 a + \cos^2 a) - (\sin^2 b + \cos^2 b)}{(\cos a + \cos b)(\sin a + \sin b)} \quad (5) \end{aligned}$$

We knew: $\sin^2 x + \cos^2 x = 1$ (6)

(6) \rightarrow (5)

$$\begin{aligned} \frac{(\sin^2 a + \cos^2 a) - (\sin^2 b + \cos^2 b)}{(\cos a + \cos b)(\sin a + \sin b)} &= \frac{1 - 1}{(\cos a + \cos b)(\sin a + \sin b)} \\ &= \frac{0}{(\cos a + \cos b)(\sin a + \sin b)} = 0 \end{aligned}$$