$$\sqrt{x^2+y^2} = \ln(x^2-y^2)$$

Solution.

Domain of the function: $x^2 - y^2 > 0$

Since the right and left side of the equation are equal, we can present it as a system of equations:

$$\begin{cases} \sqrt{x^2+y^2} = c \\ \ln(x^2-y^2) = c \end{cases}$$

So we introduced the right and left side of the equation as a constant c. This constant $c \geq 0$, because $\sqrt{x^2+y^2} \geq 0$.

Then solve this system:

$$\begin{cases} x^2+y^2 = c^2 \\ x^2-y^2 = e^c \end{cases}$$

Add these two equations:

$$2x^2 = c^2 + e^c$$

$$x = \pm\sqrt{\frac{c^2+e^c}{2}}$$

Then substract these equations:

$$2y^2 = c^2 - e^c$$

$$y = \pm\sqrt{\frac{c^2-e^c}{2}}$$

So we have solution of the equation:

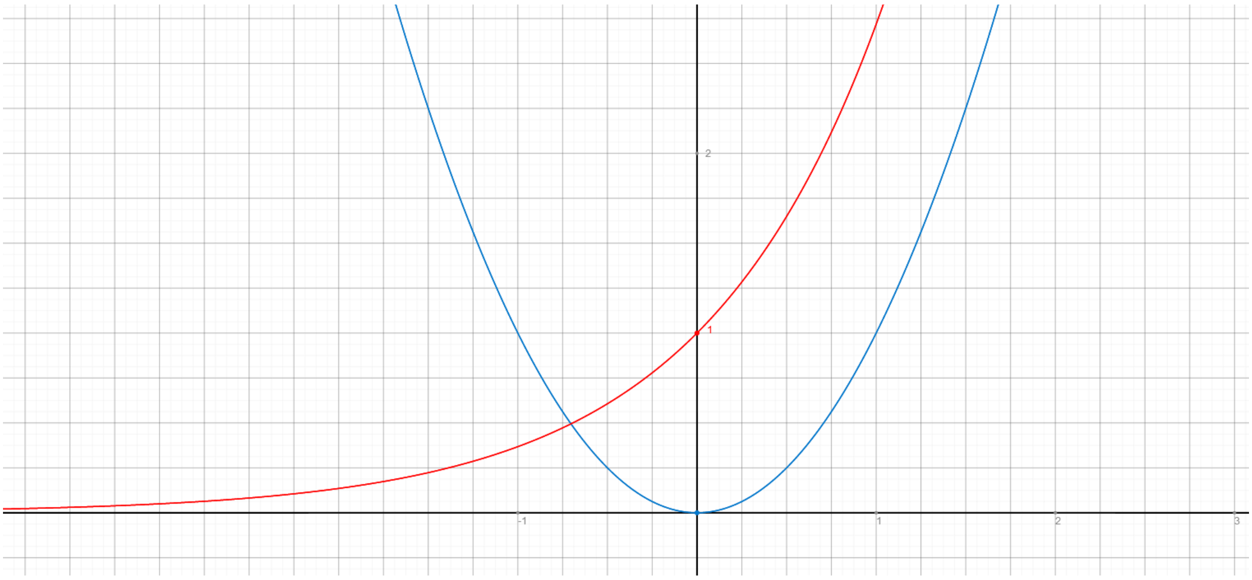
$$\begin{cases} x = \pm\sqrt{\dfrac{c^2+e^c}{2}} \\ y = \pm\sqrt{\dfrac{c^2-e^c}{2}} \end{cases}$$

Find the allowable values of c:

$$\begin{cases} \dfrac{c^2+e^c}{2} \geq 0 \\ \dfrac{c^2-e^c}{2} \geq 0 \end{cases}$$

$$\begin{cases} c \in \mathbb{R} \\ c^2 \geq e^c \end{cases}$$

Solve the inequality $c^2 \geq e^c$ graphically:



We must find a point of intersection $c^2\ and\ e^c$:

$$c \approx -0.703467$$

So

$$c \leq -0.703467$$

Answer:

$$\begin{cases} x = \pm\sqrt{\dfrac{c^2 + e^c}{2}} \\ y = \pm\sqrt{\dfrac{c^2 - e^c}{2}}, c \leq -0.703467 \end{cases}$$