Task. Suppose $\tan(A-B)=1$ and $\sec(A+B)=2/\sqrt{3}$. What is the smallest positive value of B?
Solution. The identity

$$\tan(A-B)=1$$

implies that

$$A-B=\frac{\pi}{4}+\pi k, \qquad k\in\mathbb{Z}.$$

Also, from

$$\sec(A+B)=2/\sqrt{3}$$

we get

$$\frac{1}{\cos(A+B)}=\frac{2}{\sqrt{3}}$$
$$\cos(A+B)=\frac{\sqrt{3}}{2}$$
$$A+B=\pm\frac{\pi}{6}+2\pi m, \qquad m\in\mathbb{Z}.$$

Hence

$$A+B-(A-B)=\pm\frac{\pi}{6}+2\pi m-\frac{\pi}{4}-\pi k, \qquad k,m\in\mathbb{Z}.$$
$$2B=\pm\frac{\pi}{6}+2\pi m-\frac{\pi}{4}-\pi k, \qquad k,m\in\mathbb{Z}.$$
$$B=\pm\frac{\pi}{12}+\pi m-\frac{\pi}{8}-\frac{\pi}{2}k, \qquad k,m\in\mathbb{Z}.$$
$$B=t\,\frac{\pi}{12}-\frac{\pi}{8}+\frac{\pi}{2}(2m-k), \qquad k,m\in\mathbb{Z}, t=\pm 1.$$

When (k,m) runs over all pairs of integers, the number $2m-k$ runs over all integers, therefore

$$B=t\,\frac{\pi}{12}-\frac{\pi}{8}+\frac{\pi}{2}n, \qquad n\in\mathbb{Z}, t=\pm 1.$$

Thus B depends of two parameters (n,t) and we can write it as a function

$$B(k,m,t)=t\,\frac{\pi}{12}-\frac{\pi}{8}+\frac{\pi}{2}n, \qquad n\in\mathbb{Z}, t=\pm 1.$$

We should find values of (n,t) that give the smallest positive value for B.

$$B(0,1)=\frac{\pi}{12}-\frac{\pi}{8}=\frac{2-3}{24}=-\frac{1}{24}<0,$$
$$B(0,-1)=-\frac{\pi}{12}-\frac{\pi}{8}=\frac{-2-3}{24}=-\frac{5}{24}<0,$$

Thus negative values of n will decrease B, and therefore the smallest positive value of B is achieved for positive n:

$$B(1,1)=\frac{\pi}{12}\frac{\pi}{8}+\frac{\pi}{2}=-\frac{1}{24}+\frac{\pi}{2}=\frac{-1+12}{24}=\frac{11}{24},$$
$$B(1,-1)=-\frac{\pi}{12}-\frac{\pi}{8}+\frac{\pi}{2}=-\frac{5}{24}+\frac{\pi}{2}=\frac{-5+12}{24}=\frac{7}{24}.$$

Since for fixed t the number $B(n,t)$ increases with n, the smallest positive value of B is achieved for positive $n=1$, $t=-1$, and is equal to

$$\frac{7}{24}.$$