What do you understand by divergence of a vector field? What is the meaning of positive, negative, and zero divergence?

Answer

Divergence at a point (x,y,z) is the measure of the vector flow out of a surface surrounding that point. That is, imagine a vector field represents water flow. Then if the divergence is a positive number, this means water is flowing out of the point (like a water spout - this location is considered a source). If the divergence is a negative number, then water is flowing into the point (like a water drain - this location is known as a sink).

I will give some examples to make this clearer. First, imagine we have a vector field (given by the vector function **A**) as shown in Figure 1, and we want to know what the divergence is at the point *P*:

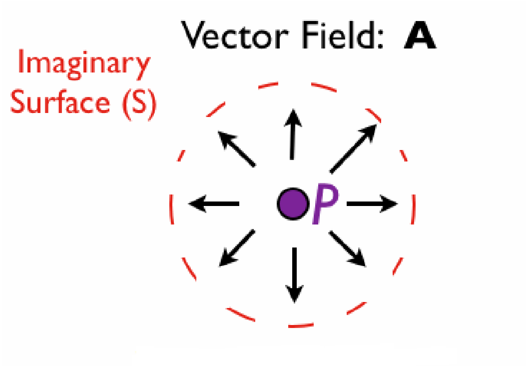

Figure 1. Example of a Vector Field Surrounding a Point.

We also draw an imaginary surface (*S*) surrounding the point *P*. Now imagine the vector **A** represents water flow. Then, if you add up the amount of water flowing out of the surface, would the amount be positive? The answer, is yes: water is flowing out of the surface at every location along the surface *S*. Hence, we can say that the divergence at *P* is positive.

Let's take another simple example, that of Figure 2. We have a new vector field **B** surrounding the point *P*:

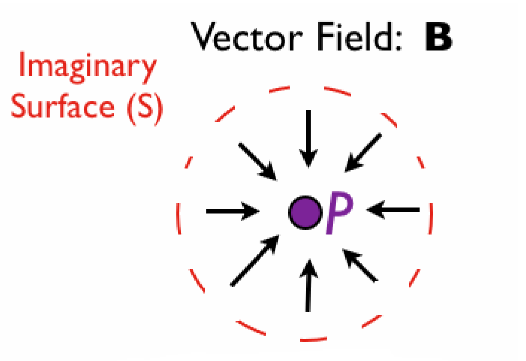

Figure 2. Example of a Vector Field Surrounding a Point (negative divergence).

In Figure 2, if we imagine the water flowing, we would see the point *P* acting like a drain or water sink. In this case, the flow *out of the surface* is negative - hence, the divergence of the field **B** at *P* is negative.

Pretty simple, eh? Here's a couple more examples. Figure 3 has a vector field **C** surrounding the point:

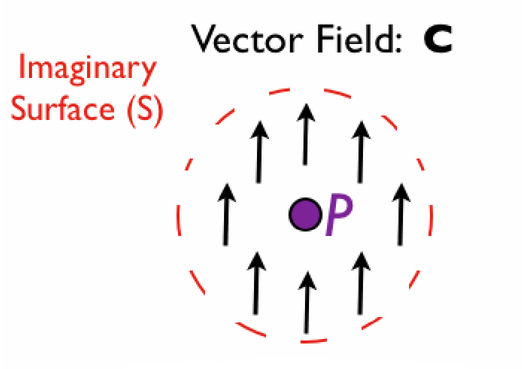

Figure 3. Example of a Vector Field with no Variation around a Point.

In Figure 3, if **C** represents the flow of water, does more water flow into or out of the surface? In the top of Figure 3, water flows out of the surface, but at the bottom it flows in. Since the field has equal flow into and out of the surface *S*, the **divergence is zero**.