By Cayley-Hamilton Theorem matrix is a root of its characteristic polynomial, so

$A^n + c_1 A^{n-1} + ... + c_{n-1} A + c_n = 0$

As it is known, $c_1 = tr(A), c_n = \det A$. If A is non-singular, then $\det A \neq 0$ and

$\left(A^{n-1} + c_1 A^{n-2} + ... + c_{n-1}\right) A = -\det A$

$\left(-\frac{1}{\det A} A^{n-1} - \frac{c_1}{\det A} A^{n-2} - ... - \frac{c_{n-1}}{\det A}\right) A = 1_n$

Thus $-\frac{1}{\det A} A^{n-1} - \frac{c_1}{\det A} A^{n-2} - ... - \frac{c_{n-1}}{\det A}$ will be inverse for A.