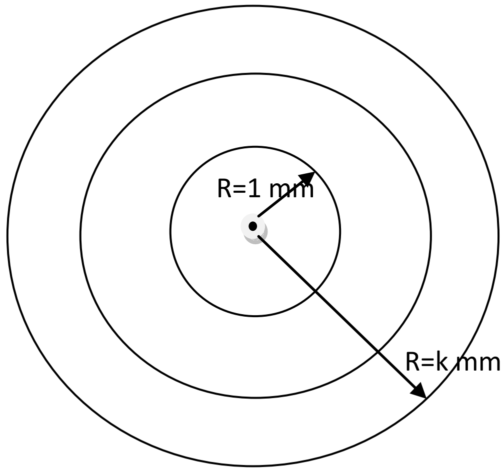
I'm doing a project for college and I've hit a stumbling block. Essentially I'm designing a large 120ft tape measure. Tape width 50mm and height 1mm. I need to find the reel size of this tape when fully retracted so that I can design the case.

Solution.

Case 1.



We have to count quantity of hanks of a tape on a reel. The reel size (radius) of this tape equals quantity of hanks (in mm). And the sum of lengths of all hanks has to equal 120 foots. Circle length is

$$S = 2\pi R.$$

Then we have

$$2\pi(1 + 2 + \cdots + k) = 120 \cdot 0{,}3048 \cdot 1000\ (mm),$$

$$2\pi \cdot \frac{1+k}{2} \cdot k = 120 \cdot 0{,}3048 \cdot 1000$$

where k is quantity of hanks of a tape on reel. In the left side of last equation we used the formula for sum of an arithmetic progression. Then we have

$$k^2 + k - 11642.50 = 0,$$

$$D = 1^2 - 4 \cdot 1 \cdot (-11642.50) = 1 + 46570 = 46571,$$

$$k_{1,2} = \frac{-1 \pm \sqrt{D}}{2}.$$

And finally

$$k = \frac{-1 + \sqrt{46571}}{2} \approx 108.$$

Then the radius of reel is 108 mm.

Case 2.

Let r (mm) is initial value of radius of a reel. Then we have

$$2\pi((1+r)+(2+r)+\cdots+(k+r)) = 120 \cdot 0{,}3048 \cdot 1000\ (mm),$$

$$2\pi \cdot \frac{1+k}{2} \cdot k + 2\pi kr = 120 \cdot 0{,}3048 \cdot 1000$$

$$k^2 + (1+2r)k - 11642.50 = 0,$$

$$D = (1+2r)^2 - 4 \cdot 1 \cdot (-11642.50) = (1+2r)^2 + 46570,$$

$$k_{1,2} = \frac{-(1+2r) \pm \sqrt{D}}{2}.$$

And finally

$$k = \frac{-(1+2r) + \sqrt{(1+2r)^2 + 46570}}{2}.$$

Then the radius of reel is $\frac{-(1+2r)+\sqrt{(1+2r)^2+46570}}{2}\ (mm)$.

Answer:

Case 1. The radius of reel is 108 mm.

Case 2. The radius of reel is $\frac{-(1+2r)+\sqrt{(1+2r)^2+46570}}{2}\ (mm)$.