

Let us have that  $I - XY \in GL_n(R)$ . Then

$$\begin{aligned}(I - YX)\left(I + Y(I - XY)^{-1}X\right) &= I + Y(I - XY)^{-1}X - YX - YXY(I - XY)^{-1}X = \\ &= I + Y(I - XY)(I - XY)^{-1}X - YX = I\end{aligned}$$

So,  $(I - YX)$  is invertible. Replacing  $X$  and  $Y$  in last formula shows reverse conclusion.