

Let $\alpha : M \rightarrow M$ be surjective and M be noetherian. The ascending chain $\ker \alpha \subseteq \ker \alpha^2 \subseteq \dots$ must stop, so $\ker \alpha^i = \ker \alpha^{i+1}$ for some i . If $\alpha(m) = 0$, write $m = \alpha^i(m')$ for some $m' \in M$. Then

$$0 = \alpha(\alpha^i(m')) = \alpha^{i+1}(m')$$

implies that $0 = \alpha^i(m') = m$, so $\alpha \in \text{Aut}(M)$.