We shall use the following crucial fact about the Jacobson radical of a ring S:
if J is an ideal in rad(S) and $\overline{S} = S/J$ then $a \in S$ has an inverse iff $\overline{a} \in \overline{S}$ does.
Then we deduce that S is Dedekind – finite iff $\overline{S}$ is.
Let $S = M_n(R)$. Then

$$J = M_n(I) \subseteq M_n(rad\ R) = rad\ S$$

$$S/J = M_n(R)/M_n(I) \cong M_n(R/I)$$

Then $S = M_n(R)$ is Dedekind – finite iff $S/J = M_n(R/I)$ is. Since this holds for all n, then R is stably finite iff R/I is.