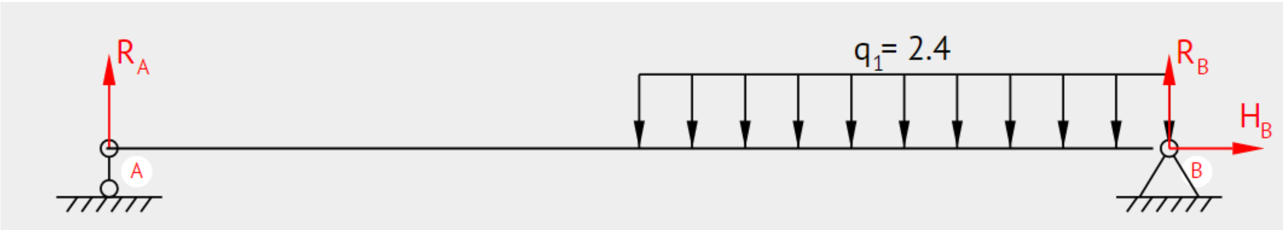
Answer on Question #64290-Engineering-Civil and Environmental Engineering

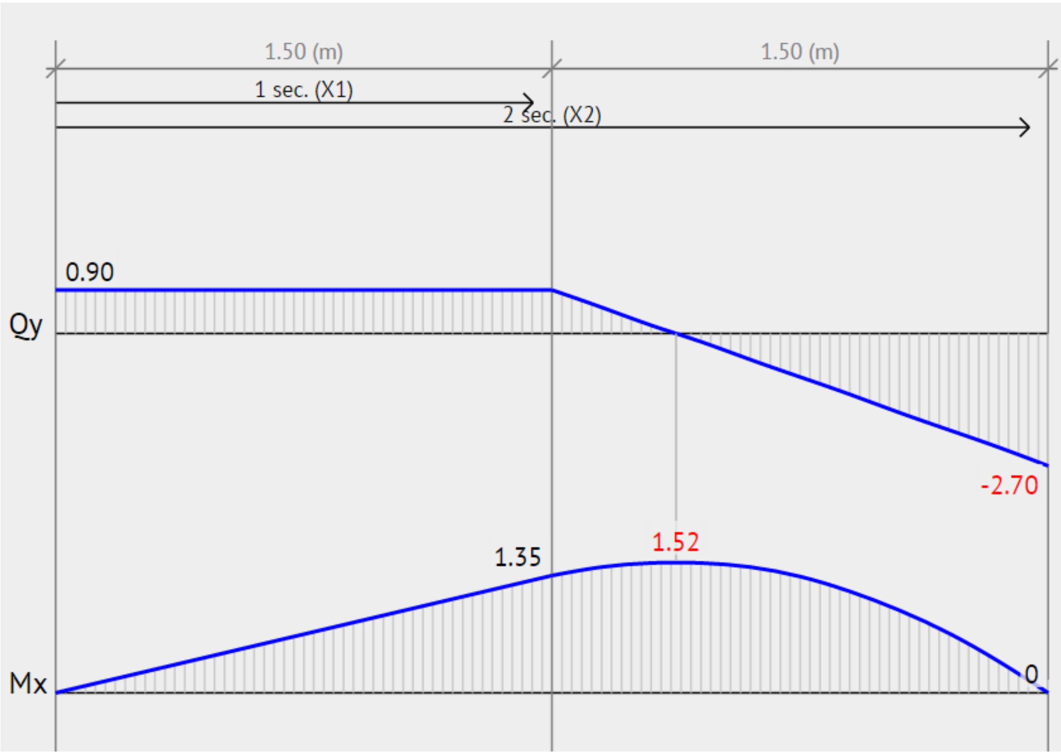
A beam, 3 m long is freely supported at its ends and carries a uniformly distributed load of 2.4 kN/m run from B to its center. Draw the shearing force and bending moment diagram and give the magnitude and position of the maximum bending moment

Solution

Free-Body-Diagram



The shearing force and bending moment diagram



The magnitude and position of the maximum bending moment: 1.52 kNm at 1.88 m.

1. A beam is in equilibrium when it is stationary relative to an inertial reference frame. The following conditions are satisfied when a beam, acted upon by a system of forces and moments, is in equilibrium:

$$\Sigma F_x = 0: \quad H_B = 0$$

$\Sigma M_A = 0$: The sum of the moments about a point A is zero:

$$-q_1 1.5\left(1.5 + \frac{1.5}{2}\right) + R_B 3 = 0$$

$\Sigma M_B = 0$: The sum of the moments about a point B is zero:

$$-R_A 3 + q_1 1.5\left(1.5 - \frac{1.5}{2}\right) = 0$$

2. Solve this system of equations:

$$H_B = 0\ (kN)$$

Calculate reaction of pin support about point B:

$$RB = \frac{q_1 1.5\left(1.5 + \frac{1.5}{2}\right)}{3} = \frac{2.4 \cdot 1.5\left(1.5 + \frac{1.5}{2}\right)}{3} = 2.70\ (kN)$$

Calculate reaction of roller support about point A:

$$RA = \frac{q_1 1.5\left(1.5 - \frac{1.5}{2}\right)}{3} = \frac{2.4 \cdot 1.5\left(1.5 - \frac{1.5}{2}\right)}{3} = 0.90\ (kN)$$

3. The sum of the forces is zero:

$$\Sigma F_y = 0: \ R_A - q_1 1.5 + R_B = 0.90 - 2.4 \cdot 1.5 + 2.70 = 0$$

Draw diagrams for the beam

First span of the beam: $0 \le x_1 < 1.5$

Determine the equations for the shear force (Q):

$$Q(x_1) = R_A$$

$$Q1(0) = 0.90 = 0.90\ (kN)$$

$$Q1(1.50) = 0.90 = 0.90\ (kN)$$

Determine the equations for the bending moment (M):

$$M(x_1) = R_A(x_1)$$

$$M_1(0) = 0.90(0) = 0\ (kNm)$$

$$M_1(1.50) = 0.90(1.50) = 1.35\ (kNm)$$

Second span of the beam: $1.5 \le x_2 < 3$

Determine the equations for the shear force (Q):

$$Q(x_2) = RA - q_1(x_2 - 1.5)$$

$$Q_2(1.50) = 0.90 - 2.40(1.5 - 1.5) = 0.90\ (kN)$$

$$Q_2(3) = 0.90 - 2.40(3 - 1.5) = -2.70\ (kN)$$

The value of Q on this span that crosses the horizontal axis. Intersection point:

$$x = 0.38$$

Determine the equations for the bending moment (M):

$$M(x_2) = R_A(x_2) - \frac{q_1(x_2 - 1.5)^2}{2}$$

$$M_2(1.50) = 0.90(1.50) - \frac{2.40(1.50 - 1.5)^2}{2} = 1.35\ (kNm)$$

$$M_2(3) = 0.90(3) - \frac{2.40(3 - 1.5)^2}{2} = 0\ (kNm)$$

Local extremum at the point x = 0.38:

$$M_2(1.88) = 0.90(1.88) - \frac{2.40(1.88 - 1.5)^2}{2} = 1.52\ (kNm)$$