Answer on Question 52114, Economics, Microeconomics

Question:

Given the total cost function:

$TC = 1000 + 10Q - 0.9Q^2 + 0.04Q^3$

Find the rate of output that results in minimum average variable costs.

Solution:

We know, that in economics total cost is made up of fixed costs plus variable costs:

$$TC = FC + VC.$$

Fixed costs are costs that don't change from month to month and don't vary based on activities or number of goods produced. Variable costs are costs that change based, for example, on how many goods we buy or how much of a service we use (VC vary with the quantity Q of output produced). According to this definition, the variable costs are:

$$VC = 10Q - 0.9Q^2 + 0.04Q^3.$$

By the definition, average variable costs is firm's variable costs divided by the quantity of output produced Q:

$$AVC = \frac{VC}{Q} = \frac{10Q - 0.9Q^2 + 0.04Q^3}{Q} = 10 - 0.9Q + 0.04Q^2.$$

We can find minimum average variable costs when the derivative of average variable costs is equal to zero:

$$\frac{d}{dQ}(AVC) = 0,$$

$$\frac{d}{dQ}(10 - 0.9Q + 0.04Q^2) = 0,$$

$$-0.9 + 0.08Q = 0,$$

$$Q = \frac{0.9}{0.08} = 11.25.$$

Answer:

The rate of output that results in minimum average variable costs is $Q = 11.25$ per unit of time.