The utility that Meredith receives by consuming food F and clothing C is given by u(F,C) = FC. Suppose that Meredith's income in 1990 is $1,200 and the prices of food and clothing are $1 per unit for each. However, by 1995 the price of food has increased to $2 and the price of clothing to $3. Let 100 represent the cost of living index for 1990. Calculate the ideal and the Laspeyres cost-of-living index for Meredith for 1995. (Hint: Meredith will spend equal amounts on food and clothing with these preferences.)

Laspeyres Index

The Laspeyres index represents how much more Meredith would have to spend in 1995 versus 1990 if she consumed the same amounts of food and clothing in 1995 as she did in 1990. That is, the Laspeyres index for 1995 (L) is given by:

$$L = 100\,(Y')/Y$$

where Y' represents the amount Meredith would spend at 1995 prices consuming the same amount of food and clothing as in 1990: $Y' = P'_F F + P'_C C = 2F + 3C$, where F and C represent the amounts of food and clothing consumed in 1990.

We thus need to calculate F and C, which make up the bundle of food and clothing which maximizes Meredith's utility given 1990 prices and her income in 1990. Use the hint to simplify the problem: Since she spends equal amounts on both goods, $P_F F = P_C C$. Or, you can derive this same equation mathematically: With this utility function, $MU_C = \Delta U/\Delta C = F$, and $MU_F = \Delta U/\Delta F = C$. To maximize utility, Meredith chooses a consumption bundle such that $MU_F/MU_C = P_F/P_C$, which again yields $P_F F = P_C C$.

From the budget constraint, we also know that:

$$P_F F + P_C C = Y.$$

Combining these two equations and substituting the values for the 1990 prices and income yields the system of equations:

$$C = F \text{ and } C + F = 1{,}200.$$

Solving these two equations, we find that:

$$C = 600 \text{ and } F = 600.$$

Therefore, the Laspeyres cost-of-living index is:

$$L = 100(2F + 3C)/Y = 100[(2)(600) + (3)(600)]/1200 = 250.$$

Ideal Index

The ideal index represents how much more Meredith would have to spend in 1995 versus 1990 if she consumed amounts of food and clothing in 1995 which would give her the same amount of utility as she had in 1990. That is, the ideal index for 1995 (I) is given by:

$$I = 100(Y'')/Y, \text{ where } Y'' = P'_F F + P'_C C' = 2F' + 3C'$$

where F' and C' are the amount of food and clothing which give Meredith the same utility as she had in 1990. F' and C' must also be such that Meredith spends the least amount of money at 1995 prices to attain the 1990 utility level.

The bundle (F',C') will be on the same indifference curve as (F,C) and the indifference curve at this point will be tangent to a budget line with slope $-(P'_F/P'_C)$, where P'_F and P'_C are the prices of food and clothing in 1995. Since Meredith spends equal amounts on the two goods, we know that 2F' = 3C'. Since this bundle lies on the same indifference curve as the bundle F = 600, C = 600, we also know that F'C' = (600)(600).

Solving for F' yields:

$$F'[(2/3)F'] = 360{,}000 \text{ or } F' = \sqrt{[(3/2)360{,}000)]} = 734.8.$$

From this, we obtain C':

$$C' = (2/3)F' = (2/3)734.8 = 489.9.$$

We can now calculate the ideal index:

$$I = 100(2F' + 3C')/Y = 100[2(734.8) + (3)(489.9)]/1200 = 244.9.$$