Answer on Question #84371 – Chemistry – Other

Task:

Solution to analysis of a sample of iron ore gave the following percentage values for the iron content 7.08, 7.21, 7.12, 7.09, 7.16, 7.14, 7.07, 7.14, 7.18, 7.11. Calculate the mean, standard deviation and coefficient of variation for the values.

Solution:

Mean or average

The mean value characterizes the "central tendency" or "location" of the data.

The first mathematical manipulation is to sum () the individual points:

$$\sum = 7.08 + 7.21 + 7.12 + 7.09 + 7.16 + 7.14 + 7.07 + 7.14 + 7.18 + 7.11 = 71.3$$
$$\sum = 71.3;$$
$$n = 10.$$

Let's calculate the mean or average, which is:

$$\overline{X} = \frac{\sum (X_i)}{n} = \frac{71.3}{10} = 7.13$$

Standard deviation

The dispersion of values about the mean is predictable and can be characterized mathematically through a series of manipulations:

- The first mathematical manipulation is to sum (Σ) the individual points and calculate the mean or average, which is 71.3 divided by 10, or 7.13 in this example.
- The second manipulation is to subtract the mean value from each control value, as shown in column B. This term, shown as Xvalue - Xbar, is called the difference score. As can be seen here, individual difference scores can be positive or negative and the sum of the difference scores is always zero.

$$1)\quad (X_1 - \overline{X}) = (7.08 - 7.13) = -0.05;$$

$$2)\quad (X_2 - \overline{X}) = (7.21 - 7.13) = 0.08;$$

$$3)\quad (X_3 - \overline{X}) = (7.12 - 7.13) = -0.01;$$

$$4)\quad (X_4 - \overline{X}) = (7.09 - 7.13) = -0.04;$$

$$5)\quad (X_5 - \overline{X}) = (7.16 - 7.13) = 0.03;$$

$$6)\quad (X_6 - \overline{X}) = (7.14 - 7.13) = 0.01;$$

$$7)\quad (X_7 - \overline{X}) = (7.07 - 7.13) = -0.06;$$

$$8)\quad (X_8 - \overline{X}) = (7.14 - 7.13) = 0.01;$$

$$9)\quad (X_9 - \overline{X}) = (7.18 - 7.13) = 0.05;$$

$$10)\quad (X_{10} - \overline{X}) = (7.11 - 7.13) = -0.02$$

- The third manipulation is to square the difference score to make all the terms positive.

$$1)\quad (X_1 - \overline{X})^2 = (-0.05)^2 = 0.0025;$$

$$2)\quad (X_2 - \overline{X})^2 = 0.08^2 = 0.0064;$$

$$3)\quad (X_3 - \overline{X})^2 = (-0.01)^2 = 0.0001;$$

$$4)\quad (X_4 - \overline{X})^2 = (-0.04)^2 = 0.0016;$$

$$5)\quad (X_5 - \overline{X})^2 = 0.03^2 = 0.0009;$$

$$6)\quad (X_6 - \overline{X})^2 = 0.01^2 = 0.0001;$$

$$7)\quad (X_7 - \overline{X})^2 = (-0.06)^2 = 0.0036;$$

$$8)\quad (X_8 - \overline{X})^2 = 0.01^2 = 0.0001;$$

$$9)\quad (X_9 - \overline{X})^2 = 0.05^2 = 0.0025;$$

$$10)\quad (X_{10} - \overline{X})^2 = (-0.02)^2 = 0.0004$$

- Next the squared difference scores are summed.

$$\sum_{i=1}^{10} (X_i - \overline{X})^2 = 0.0025 + 0.0064 + 0.0001 + 0.0016 + 0.0009 + 0.0001 + 0.0036 + 0.0001 + 0.0025 + 0.0004 = 0.0182$$

$$\sum_{i=1}^{10} (X_i - \overline{X})^2 = 0.0182$$

- Finally, the predictable dispersion or standard deviation (S_D or s) can be calculated as follows:

$$S_D = \sqrt{\frac{\sum_{i=1}^{10}(X_i - \overline{X})^2}{(n-1)}}$$

$$S_D = \sqrt{\frac{\sum_{i=1}^{10}(X_i - \overline{X})^2}{(n-1)}} = \sqrt{\frac{0.0182}{(10-1)}} = \sqrt{\frac{0.0182}{9}} = \sqrt{0.002022} = 0.04497 = 0.045$$

$$S_D = 0.045$$

Coefficient of variation

The C_V expresses the variation as a percentage of the mean, and is calculated as follows:

$$C_V\% = \frac{S_D}{\overline{X}} * 100$$

$$C_V\% = \frac{S_D}{\overline{X}} * 100 = \frac{0.045}{7.13} * 100 = 0.6311\%$$

$$C_V\% = 0.6311\%$$

Answer: The mean = X_{bar} = 7.13; The standard deviation = S_D = 0.045; The coefficient of variation = C_V% = 0.6311%.