NConsider the reaction to produce ammonia shown below.

N2 (g) + 3H2 (g) *) 2NH3 (g) KP = 44 at 473 K

Initially a mixture of N2, H2, and NH3 have the following pressures.

PN2 = 10atm PH2 = 30atm PNH3 = 3atm

What is the pressure of each gas once equilibrium is established?

Solution

N_2 (g) + 3H_2 (g) = 2NH_3 (g)

$$Kp = \frac{P_{NH3}^2}{P_{N2} \cdot P_{H2}^3};$$

where P_{NH3}, P_{N2} , P_{H2} pressures of NH_3, N_2 and H_2 when equilibrium is established.

Let x be the change in pressure for N_2. Then the change in pressure for H_2 is 3x (we can see from the chemical equation that the coefficient ratio of N_2 and H_2 is 1:3), the change in pressure of NH_3 is 2x (we can see from the chemical equation that the coefficient ratio of N_2 and NH_3 is 1:2).

Let's express pressures of gases when equilibrium is established in the form of table:

	N_2, atm	H_2, atm	NH_3,atm
Initial pressure	10	30	3
Change in pressure	-x	-3x	+2x
Pressure of gases when equilibrium is established	10-x	30-3x	3+2x

Comment: for N_2 and H_2 change in pressure is negative as pressures of these gases decrease, for NH_3 change in pressure is positive as it's pressure increase.

Solve the equation and find x:

$$Kp = \frac{P_{NH3}^2}{P_{N2} \cdot P_{H2}^3};$$

$$44 = \frac{(3+2x)^2}{(10-x)\cdot(30-3x)^3};$$

$44(10-x)\cdot(30-3x)^3= (3+2x)^2$;

$44\ (10-x)\cdot3^3(10-x)^3 = (3+2x)^2$;

$1188(10-x)^4 - (3+2x)^2 = 0$;

$34.47^2(10-x)^4 - (3+2x)^2=0$;

$(34.47(10-x)^2 – (3+2x)) \cdot(34.47(10-x)^2 + (3+2x))= 0$;

$34.47(10-x)^2 – (3+2x)= 0$ or $34.47(10-x)^2 + (3+2x) =0$.

Find roots of the first equation:

$34.47(10-x)^2 – (3+2x)=0$;

$34.47(100-20x+x^2) -3-2x =0$;

$3447- 689.4x+34.47x^2 -3-2x =0$

$34.47x^2 -691.4x +3444=0$

$D = (-691.4)^2 -4\cdot34.47\cdot3444 = 478033.96 – 474858.72 =3175.24 =56.35^2$;

$x_1= (691.4-56.35)/2\cdot34.47 = 9.21$;

$x_2 = (691.4+56.35)/2\cdot34.47= 10.85$.

Find roots of the second equation:

$34.47(10-x)^2 + (3+2x) =0$;

$34.47(100-20x+x^2) +3+2x =0$;

$3447- 689.4x+34.47x^2 +3+2x =0$

$34.47x^2 -687.4x +3450=0$

$D = (-687.4)^2 -4\cdot34.47\cdot3450 = 472518.76 – 475686 = -3167.24$;

$D<0$, the second equation has no roots.

Analysis of the roots: $x_1= 9.21$; $x_2 = 10.85$. Initial pressure of N_2 10 atm, then P_{N2} when equilibrium is established should be positive: $P_{N2} = 10-x= 10-9.21= 0.79$ (atm)(compare: 10-10.85 = -0.85). We should use x=9.21.

$P_{H2}= 30-3x= 30-3\cdot9.21=2.37$ (atm).

$P_{NH3} = 3+2x = 3+2\cdot9.21 =21.42$ (atm).

Answer: $P_{N2} = 0.79$ atm, $P_{H2}= 2.37$ atm, $P_{NH3} = 21.42$ atm.