

Consider the reaction to produce ammonia shown below.

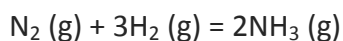


Initially a mixture of N_2 , H_2 , and NH_3 have the following pressures.

$$P_{\text{N}_2} = 10 \text{ atm} \quad P_{\text{H}_2} = 30 \text{ atm} \quad P_{\text{NH}_3} = 3 \text{ atm}$$

What is the pressure of each gas once equilibrium is established?

Solution



$$K_p = \frac{P_{\text{NH}_3}^2}{P_{\text{N}_2} \cdot P_{\text{H}_2}^3};$$

where P_{NH_3} , P_{N_2} , P_{H_2} pressures of NH_3 , N_2 and H_2 when equilibrium is established.

Let x be the change in pressure for N_2 . Then the change in pressure for H_2 is $3x$ (we can see from the chemical equation that the coefficient ratio of N_2 and H_2 is 1:3), the change in pressure of NH_3 is $2x$ (we can see from the chemical equation that the coefficient ratio of N_2 and NH_3 is 1:2).

Let's express pressures of gases when equilibrium is established in the form of table:

	N_2 , atm	H_2 , atm	NH_3 , atm
Initial pressure	10	30	3
Change in pressure	-x	-3x	+2x
Pressure of gases when equilibrium is established	10-x	30-3x	3+2x

Comment: for N_2 and H_2 change in pressure is negative as pressures of these gases decrease, for NH_3 change in pressure is positive as its pressure increase.

Solve the equation and find x :

$$K_p = \frac{P_{\text{NH}_3}^2}{P_{\text{N}_2} \cdot P_{\text{H}_2}^3};$$

$$44 = \frac{(3+2x)^2}{(10-x) \cdot (30-3x)^3};$$

$$44(10-x) \cdot (30-3x)^3 = (3+2x)^2;$$

$$44(10-x) \cdot 3^3(10-x)^3 = (3+2x)^2;$$

$$1188(10-x)^4 - (3+2x)^2 = 0;$$

$$34.47^2(10-x)^4 - (3+2x)^2 = 0;$$

$$(34.47(10-x)^2 - (3+2x)) \cdot (34.47(10-x)^2 + (3+2x)) = 0;$$

$$34.47(10-x)^2 - (3+2x) = 0 \text{ or } 34.47(10-x)^2 + (3+2x) = 0.$$

Find roots of the first equation:

$$34.47(10-x)^2 - (3+2x) = 0;$$

$$34.47(100-20x+x^2) - 3 - 2x = 0;$$

$$3447 - 689.4x + 34.47x^2 - 3 - 2x = 0$$

$$34.47x^2 - 691.4x + 3444 = 0$$

$$D = (-691.4)^2 - 4 \cdot 34.47 \cdot 3444 = 478033.96 - 474858.72 = 3175.24 = 56.35^2;$$

$$x_1 = (691.4 - 56.35) / 2 \cdot 34.47 = 9.21;$$

$$x_2 = (691.4 + 56.35) / 2 \cdot 34.47 = 10.85.$$

Find roots of the second equation:

$$34.47(10-x)^2 + (3+2x) = 0;$$

$$34.47(100-20x+x^2) + 3 + 2x = 0;$$

$$3447 - 689.4x + 34.47x^2 + 3 + 2x = 0$$

$$34.47x^2 - 687.4x + 3450 = 0$$

$$D = (-687.4)^2 - 4 \cdot 34.47 \cdot 3450 = 472518.76 - 475686 = -3167.24;$$

$D < 0$, the second equation has no roots.

Analysis of the roots: $x_1 = 9.21$; $x_2 = 10.85$. Initial pressure of N_2 10 atm, then P_{N_2} when equilibrium is established should be positive: $P_{N_2} = 10 - x = 10 - 9.21 = 0.79$ (atm) (compare: $10 - 10.85 = -0.85$). We should use $x = 9.21$.

$$P_{H_2} = 30 - 3x = 30 - 3 \cdot 9.21 = 2.37 \text{ (atm)}.$$

$$P_{NH_3} = 3 + 2x = 3 + 2 \cdot 9.21 = 21.42 \text{ (atm)}.$$

Answer: $P_{N_2} = 0.79$ atm, $P_{H_2} = 2.37$ atm, $P_{NH_3} = 21.42$ atm.