Answer on Question #73627, Chemistry / General Chemistry:

Show that for a cyclic process involving isothermal reversible expansion and reversible compression, the total entropy change is zero

Solution.

$T = const$

$\Delta S_T - ?$

Entropy is typically considered a function of temperature and either volume or pressure. When we hold temperature constant (an isothermal process), and change one of the other parameters:

$$\Delta S = S_2 - S_1 = \int \frac{\partial Q}{T} = \int \frac{dU + \partial A}{T}$$

And:

$$dU = \frac{m}{M} C_V dT = 0$$

$$\partial A = pdV = \frac{m}{M} \cdot \frac{RT}{V} dV$$

Entropy:

$$\Delta S = S_2 - S_1 = \int \frac{\partial Q}{T} = \int \frac{\frac{m}{M} C_V dT + \frac{m}{M} \cdot \frac{RT}{V} dV}{T}$$

$$\Delta S = \frac{m}{M} C_V \cdot ln \frac{T_2}{T_1} + \frac{m}{M} R \cdot ln \frac{V_2}{V_1}$$

For a cyclic process involving isothermal reversible expansion and reversible compression $T_1 = T_2$, $V_2 = V_1$, and:

$$\Delta S = \frac{m}{M} C_V \cdot ln \frac{T_2}{T_1} + \frac{m}{M} R \cdot ln \frac{V_2}{V_1}$$

$$\Delta S = \frac{m}{M} C_V \cdot ln 1 + \frac{m}{M} R \cdot ln 1 = 0$$

Answer: $\Delta S_T = 0$.