Answer on Question #71623 - Chemistry - General Chemistry

Question:
Using the method of successive approximations, what is [H3O+] in 0.00500 M CH2ClCOOH(aq) (Ka=1.4×10–3) and how many iterations (n) are needed to validate that a constant value had been obtained to two significant figures?

Answer:
Let's write the reaction equation:

$$CH_2ClCOOH + H_2O \leftrightarrow H_3O^+ + CH_2ClCOO^-$$

Thus, the deprotonation constant is:

$$K_a = \frac{[H_3O^+][CH_2ClCOO^-]}{[CH_2ClCOOH]} = \frac{x^2}{0.00500 - x}$$

At first step, we assume that x that is equal to $[H_3O^+]$ and to $[CH_2ClCOO^-]$, is much less than the total concentration of the acid, 0.00500M. So, we can neglect x in the denominator and get :

$$x = \sqrt{0.00500 \cdot 1.4 \cdot 10^{-3}} = 0.002646\ M,$$

Then, the next iteration will be such that we feed the value of x to determine the new x.

$$x_n = \sqrt{K_a(0.00500 - x_{n-1})}$$

First iteration :

$$x_1 = \sqrt{K_a(0.00500 - 0.002646)} = 0.001815\ M$$

Second iteration :

$$x_2 = \sqrt{K_a(0.00500 - 0.001815)} = 0.002111\ M$$

Third iteration :

$$x_3 = \sqrt{K_a(0.00500 - 0.002111)} = 0.002011\ M$$

Fourth iteration :

$$x_4 = \sqrt{K_a(0.00500 - 0.002011)} = 0.002046\ M$$

As one can see, fourth iterations already gives the constant value when only two significant figures are considered ($x_3 = x_5 = 0.0020\ M$).

Really, the successive approximation method is computationally faster than the resolution of second order equation.