Answer on the question #69581, Chemistry / Physical Chemistry

Question:

One mol of an ideal gas with initial volume of 1m3 at 50K & 2bar undergoes a four steps mechanically reversible cycle in a closed system?

In step 1-2 the volume is increasing to 5m3 at a constant pressure of 2bar; in step 2-3, pressure decreases at constant volume to 1 bar and a temp of 100 K in step 3-4 the pressure increases to 10 bar by an isothermal process; and in step 4-1 the gas return by constant volume process to initial state. * Calculate Q, W, ΔU, and ΔH for each step of the cycle. Take Cv =3/2 R; CP=5/2 R

Answer:

Applying ideal gas law, we can check the initial state:

$$pV = nRT$$

$$2bar \cdot 1m^3 = 1mol \cdot 8.314 \frac{J}{mol \cdot K} \cdot 50K$$

$$2 \cdot 10^5 Pa \cdot 1m^3 = 1mol \cdot 8.314 \frac{J}{mol \cdot K} \cdot 50K$$

$$2 \cdot 10^5 J = 415.7\,J$$

Obviously, there is a mistake in the initial state of the system. However, we can still solve the problem, assuming 200000/415.7 mol of ideal gas:

$$\boldsymbol{n = 481.1\ mol}$$

The **first step** 1-2 of the cycle is isobaric expansion:

$$Q = nC_P\Delta T = 481.1\ mol \cdot \frac{5}{2} \cdot 8.314 \frac{J}{K \cdot mol} \cdot \Delta T$$

Let's find the change in temperature:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}\ ; T_2 = \frac{V_2 \cdot T_1}{V_1} = \frac{5m^3 \cdot 50K}{1m^3} = 250\ K$$

$$\Delta T = 250\ K - 50K = 200\ K$$

Thus, heat absorbed by the system is:

$$Q = 2 \cdot 10^6 J$$

The work of the process is:

$$W = -\int_{V_1}^{V_2} PdV = -2 \cdot 10^5 Pa \cdot (5 - 1\ m^3) = -8 \cdot 10^5 J$$

Change of internal energy:

$$\Delta U = Q + W = 2 \cdot 10^6 J\ - 8 \cdot 10^5 J = 1.2\ \cdot 10^6 J$$

In the **step 2-3** isochoric compression of gas takes place:

$$Q = nC_V\Delta T = 481.1\,mol \cdot \frac{3}{2} \cdot 8.314\frac{J}{mol \cdot K} \cdot \Delta T$$

Let's find change in temperature:

$$\frac{p_2}{T_2} = \frac{p_3}{T_3};T_3 = \frac{p_3 \cdot T_2}{p_2} = \frac{1bar \cdot 250K}{2\,bar} = 125\,K$$

Finally, we see one more mistake in the data given (100 K instead of 125K).

$$Q = 481.1\,mol \cdot \frac{3}{2} \cdot 8.314\frac{J}{mol \cdot K} \cdot (125\,K - 250K) = -7.5 \cdot 10^5 J$$

Work is equal to zero, as the volume stays constant. Thus, change of internal energy is:

$$W = 0; \Delta U = Q + W = -7.5 \cdot 10^5 J$$

Change in enthalpy is:

$$\Delta H = Q + V\Delta p = -7.5 \cdot 10^5 J + 5m^3 \cdot (2-1) \cdot 10^5 Pa = -2.5 \cdot 10^5 J$$

Step 3-4 is isothermal process, where the change of internal energy is zero:

$$\Delta U = 0$$

For that, the heat is equal to the work:

$$Q = -W = nRT\ln\frac{V_4}{V_3} = 481.1\,mol \cdot 8.314\frac{J}{mol \cdot K} \cdot 125K \cdot \ln\frac{1}{5} = 8.05 \cdot 10^5 J$$

Then, change in enthalpy for isothermal process is also zero:

$$\Delta H = 0$$

Task says that the pressure in point 3 is 10 bar. But, there is a mistake:

$$\boldsymbol{p_4V_4 = p_3V_3\,;p_4 = \frac{p_3V_3}{V_4} = \frac{1bar \cdot 5m^3}{1m^3} = 5bar}$$

Step 4-1 is one more isochoric compression:

$$Q = nC_V\Delta T;$$
$$\Delta T = T_4 - T_3 = 50K - 125K = -75K$$
$$Q = 481.1\,mol \cdot \frac{3}{2} \cdot 8.314\frac{J}{mol \cdot K} \cdot (-75K) = -4.5 \cdot 10^5 J$$

Work is equal to zero, as the volume stays constant. Thus, change of internal energy is:

$$W = 0; \Delta U = Q + W = -4.5 \cdot 10^5 J$$

Change in enthalpy is:

$$\Delta H = Q + V\Delta p = -4.5 \cdot 10^5 J + 1m^3 \cdot (5-1) \cdot 10^5 Pa = -0.5 \cdot 10^5 J$$