Answer on the question #67063, Chemistry / Physical Chemistry

Question:

The half-life for the first-order decomposition of N2O5 is 2.05 x 104 s. How long will it take for a sample of this compound to decay to 80% of its initial value?

Solution:

The equation for integrated first order rate law is

$$\ln[N_2O_5]_t = -kt + \ln[N_2O_5]_0,$$

where $[N_2O_5]_t$ and $[N_2O_5]_0$ are the concentration of N_2O_5 at time t and at time t=0 (or initial concentration) and k is the rate constant of this reaction.
We can derive the expression of relation between the half-life time and rate constant $\tau_{1/2}$. As one can guess, half-life time is the time when the initial concentration decreases two times:

$$[N_2O_5]_t = \frac{[N_2O_5]_0}{2}.$$

Introducing this relation into our equation, we get:

$$\ln\frac{[N_2O_5]_0}{2} = -k\tau_{1/2} + \ln[N_2O_5]_0$$

$$\ln\frac{[N_2O_5]_0}{2} - \ln[N_2O_5]_0 = -k\tau_{1/2}.$$

Now, using the properties of logarithm, we solve this to derive the rate constant and half-life time relation:

$$\ln\frac{[N_2O_5]_0}{2[N_2O_5]_0} = -k\tau_{1/2}$$

$$\ln\frac{1}{2} = -k\tau_{1/2}$$

$$k = \frac{\ln 2}{\tau_{1/2}}.$$

Finally, to know how long it will take for the concentration to decay to 80% of its initial value:

$$[N_2O_5]_t = 0.8 \cdot [N_2O_5]_0$$

$$\ln(0.8 \cdot [N_2O_5]_0) = -kt + \ln[N_2O_5]_0$$

$$\ln(0.8 \cdot [N_2O_5]_0) = -\frac{\ln 2}{\tau_{1/2}}t + \ln[N_2O_5]_0$$

$$\ln(0.8 \cdot [N_2O_5]_0) - \ln[N_2O_5]_0 = -\frac{\ln 2}{\tau_{1/2}}t$$

$$\ln 0.8 = -\frac{\ln 2}{\tau_{1/2}}t$$

$$t = -\frac{\ln 0.8}{\ln 2}\tau_{1/2} = -\frac{\ln 0.8}{\ln 2} \cdot 2.05 \cdot 10^4 (s) = 6600\ s$$

Answer: It will take 6600 s, or 110 minutes to decay the concentration to 80% of its initial value.