Sample: Statistics and Probability - Statistics Charts

Question 1

Parts manufactured by an injection molding process are subjected to a compressive strength test. Twenty samples of five parts each are collected and the compressive strengths (in psi) are shown in the table that follows.

x_1	x_2	x_3	x_4	x_5	$\bar{x}$	R
83.0	81.2	78.7	75.7	77.0	79.12	7.3
88.6	78.3	78.8	71.0	84.2	80.18	17.6
85.7	75.8	84.3	75.2	81.0	80.40	10.5
80.8	74.4	82.5	74.1	75.7	77.50	8.4
83.4	78.4	82.6	78.2	78.9	80.30	5.2
75.3	79.9	87.3	89.7	81.8	82.80	14.4
74.5	78.0	80.8	73.4	79.7	77.28	7.4
79.2	84.4	81.5	86.0	74.5	81.12	11.5
80.5	86.2	76.2	84.1	80.2	81.44	10.0
75.7	75.2	71.1	82.1	74.3	75.68	11.0
80.0	81.5	78.4	73.8	78.1	78.36	7.7
80.6	81.8	79.3	73.8	81.7	79.44	8.0
82.7	81.3	79.1	82.0	79.5	80.92	3.6
79.2	74.9	78.6	77.7	75.3	77.14	4.3
85.5	82.1	82.8	73.4	71.7	79.10	13.8
78.8	79.6	80.2	79.1	80.8	79.70	2.0
82.1	78.2	75.5	78.2	82.1	79.22	6.6
84.5	76.9	83.5	81.2	79.2	81.06	7.6
79.0	77.8	81.2	84.4	81.6	80.80	6.6
84.5	73.1	78.6	78.7	80.6	79.10	11.4

(a) Is there evidence to support the claim that compressive strength is normally distributed? [If your statistical package does not have a subprogram that allows you to answer this, you can briefly explain how normality can be verified or tested.]

(b) Construct $\overline{X}$ and R charts. Is the process in control?

(c) After establishing the control charts in part (b) above, 15 new subgroups were collected and the compressive strengths are shown below. Plot the $\overline{X}$ and R values against the control limits from part (b). What do the charts indicate regarding the process mean and variability?

x_1	x_2	x_3	x_4	x_5	$\bar{x}$	R
68.9	81.5	78.2	80.8	81.5	78.18	12.6
69.8	68.6	80.4	84.3	83.9	77.40	15.7
78.5	85.2	78.4	80.3	81.7	80.82	6.8
76.9	86.1	86.9	94.4	83.9	85.64	17.5
93.6	81.6	87.8	79.6	71.0	82.72	22.6
65.5	86.8	72.4	82.6	71.4	75.74	21.3
78.1	65.7	83.7	93.7	93.4	82.92	28.0
74.9	72.6	81.6	87.2	72.7	77.80	14.6
78.1	77.1	67.0	75.7	76.8	74.94	11.1
78.7	85.4	77.7	90.7	76.7	81.84	14.0
85.0	60.2	68.5	71.7	82.4	73.56	24.8
86.4	79.2	79.8	86.0	75.4	81.36	11.0
78.5	99.0	78.3	71.4	81.8	81.80	27.6
68.8	62.0	82.0	77.5	76.1	73.28	20.0
83.0	83.7	73.1	82.2	95.3	83.46	22.2

Solution

(a) To test the data for normality, the chi-square test can be used. Using the data from columns x1 – x5 construct a frequency table. Find minimum and maximum values in the data given, divide the range in 10 equal intervals, add the intervals “less than minimum” and “more than maximum”, calculate number of data points in each interval (these are observed frequencies). Then, using the normal distribution functions, find the expected frequency for each interval. The result is shown in the table below.

Class number	lower bound	upper bound	observed fr	expected fr
1		71	0	1.347291441
2	71	72.87	3	2.856805248
3	72.87	74.74	10	6.495681655
4	74.74	76.61	11	11.7278795
5	76.61	78.48	12	16.81480185
6	78.48	80.35	21	19.14512314
7	80.35	82.22	22	17.3111046
8	82.22	84.09	7	12.43045149
9	84.09	85.96	9	7.088089555
10	85.96	87.83	3	3.209416497
11	87.83	89.7	1	1.15383145
12	89.7		1	0.419523569

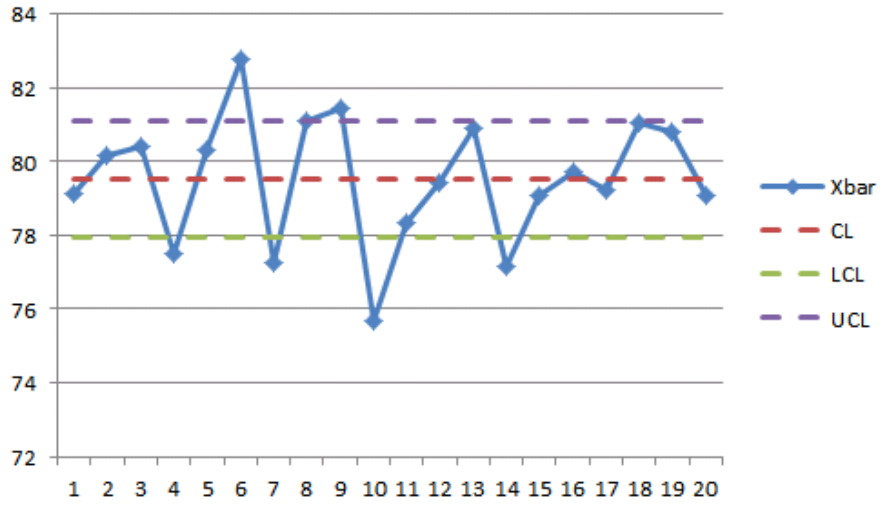
Now, use the following formula for the test statistics:

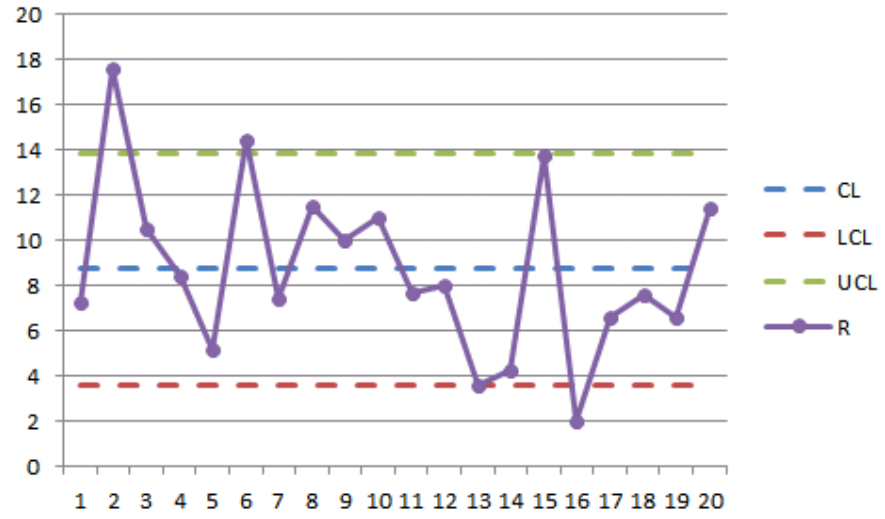
$$chi - sq = \sum \frac{(E-O)^2}{E}$$

When calculated, the test statistics can be used to determine p- value (degrees of freedom equals to number of classes minus one – in our case it is 11).

For the data given the p-value calculated is near 0.54. It is much more than the usual levels of significance (0.05 or 0.01). Thus, the null hypothesis should be rejected – the compressive strength is not normally distributed.

(b) The control Xbar and R charts are shown below. As we can see, both charts contain points above the upper control limit line and under the lower control limit line. Thus, the process is not in control.

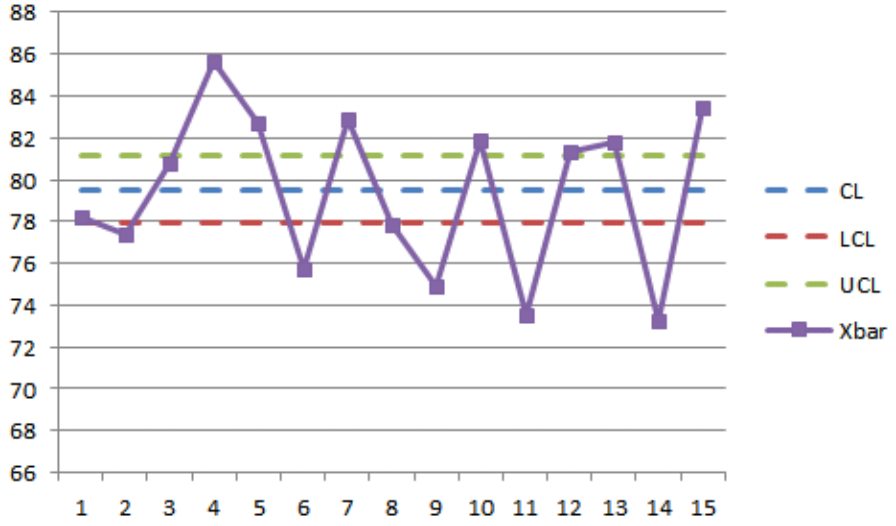


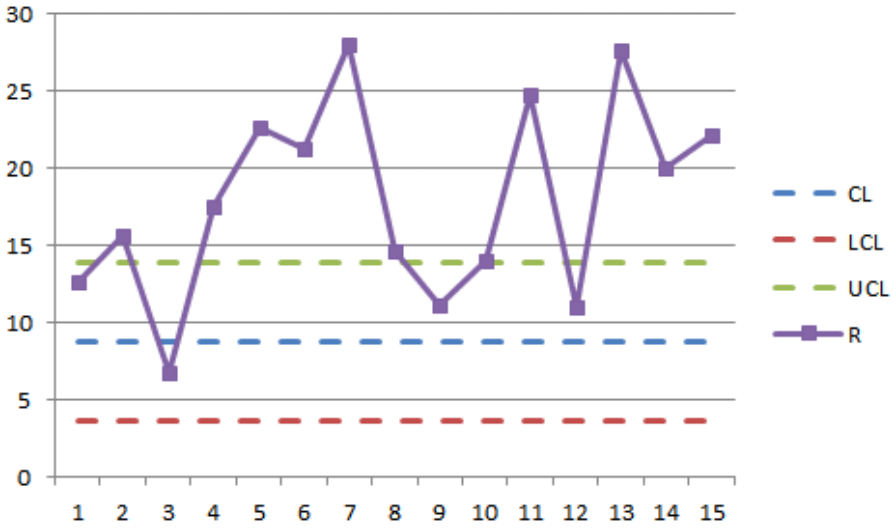


(c) The control Xbar and R charts are shown below. As we can see, Xbar chart contains points above the upper control limit line and under the lower control limit line, the R chart contains points above the UCL. Thus, the process is not in control.

Moreover, when comparing the charts with corresponding ones from part (b) one can notice that number of points on Xbar chart outside the control limits has increased and the points on R chart shifted “up”, so the average range increased. Thus, we can conclude that variability of the process has increased.

When looking at Xbar chart we can see the points locate approximately symmetrically with respect to central line. Thus, the process mean did not change significantly.





Question 2

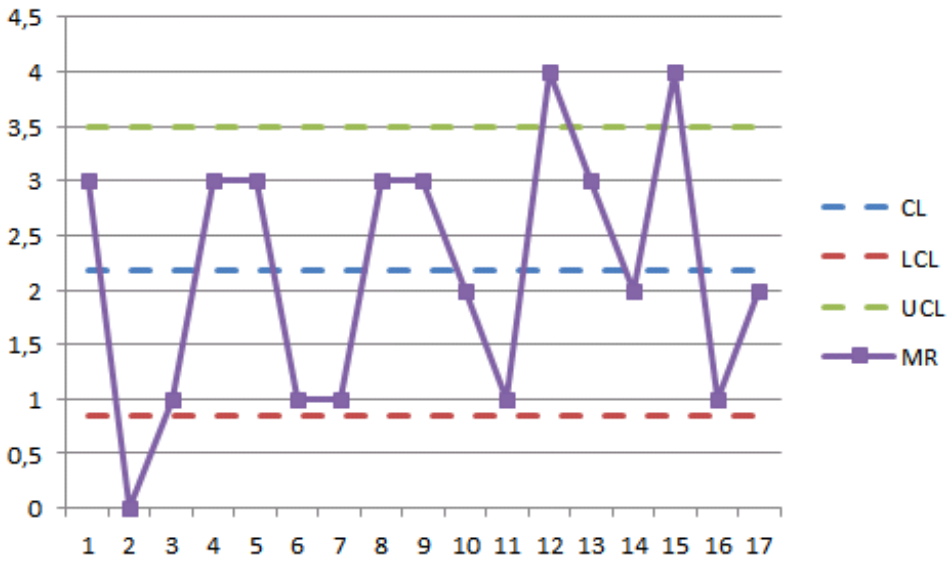
In an attempt to monitor the effectiveness of a crime prevention program, a local police chief has chosen to use a quality control chart. Since implementation of the program 18 months ago, the police chief has recorded the number of reported cases of assault and armed robbery within his jurisdiction. The following are the monthly recorded figures:

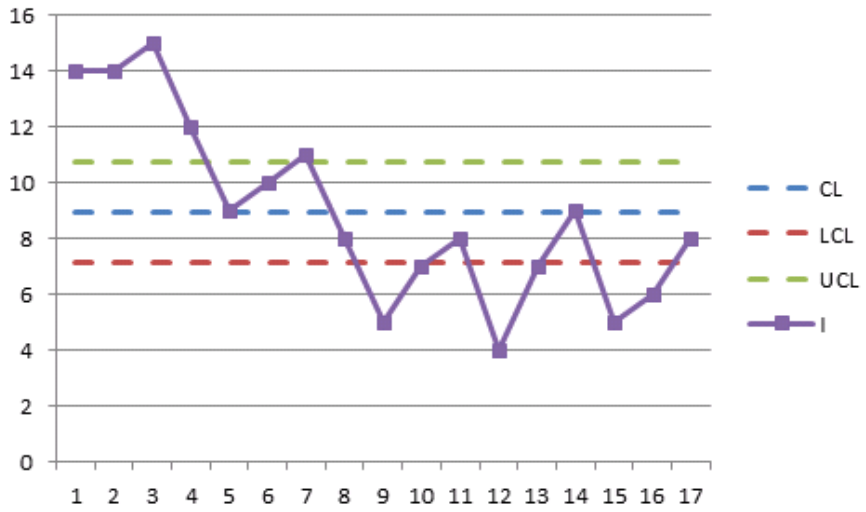
11 14 14 15 12 9 10 11 8 5 7 8 4 7 9 5 6 8

(a) State the random variable of interest and the assumption on its distribution if I-MR control charts are used to monitor the crime statistics over the review period. Plot these charts.

(b) What do the graphs tell you about the effectiveness of the program? Include 'nine points in a row on the same side of the center line' and 'six points in a row, all increasing or decreasing' signals to assess statistical control of the process.

(a) The random variable of interest for the I-MR chart is difference between the current and previous week numbers of reported cases of assault and armed robbery. The differences are assumed to be normally distributed. The control charts are shown below.





(b) The I – chart indicates that the program is effective: at the beginning of the program we can see six points in a row above the central line and in a second part of the sample under consideration 9 points in a row are below the central line. Thus, the data indicates a clear trend: numbers of reported cases of assault and armed robbery decrease. However, the MR chart contains values above and below the control limits. Thus, the decreasing of number of reported cases of assault and armed robbery is not in statistical control – we cannot predict the next week number based on this week number with a good accuracy using statistical methods.