Sample: Molecular Physics Thermodynamics - Molecular Physics Assignment

1) A helium weather balloon is used to lift a meteorological sensor package high up into the atmosphere.

At Earth's surface balloon contains 3 m^3 of helium at a pressure of 1 atmosphere. At the maximum height that the balloon reaches (≈20 km), the volume of helium in the balloon expands to 42 m^3. The temperature at the meteorological station where the balloon was released was 30 °C and at the balloon's maximum height, -56 °C.

a) Calculate the number of moles of helium in the balloon.

b) Find the atmospheric pressure the balloon experiences at its maximum height.

Solution:

a) According to the ideal gas law: $P \cdot V = n \cdot R_0 \cdot T$;

where P, V, T are the pressure, volume and absolute temperature of the gas, Pa, m^3 and K, respectively;

n is the amount of substance of gas, mol;

R_0 is the universal gas constant, 8.314 J/(mol·K).

Then, $n = \frac{P \cdot V}{R_0 \cdot T} = \frac{101{,}325 \cdot 3}{8.314 \cdot (30 + 273)} = 120.67$ mol.

b) According to the combined gas law, $\frac{P \cdot V}{T} = \frac{P_1 \cdot V_1}{T_1}$,

where P, V, T and P_1, V_1, T_1 are the pressure, volume and absolute temperature of the helium at the Earth's surface and at balloon's maximum height, Pa, m^3 and K, respectively.

We assume that atmospheric pressure outside the balloon is equal to the pressure of helium inside the balloon.

Then, $P_1 = \frac{P \cdot V \cdot T_1}{T \cdot V_1} = \frac{101{,}325 \cdot 3 \cdot (273 - 56)}{(273 + 30) \cdot 42} = 5183.3$ Pa.

Answer: a) 120.67 mol; b) 5183.3 Pa.

2) In James' research lab there is a 1x1x1 m vacuum chamber (basically stainless steel box) that contains nitrogen gas at a pressure of 1x10^{-6} Pa and a temperature of 300 K. On average is a given

nitrogen molecule in the chamber more likely to hit the walls of the chamber or another nitrogen molecule?
(Hint: think about what the mean free path of the molecules is).

Solution:

The mean free path is the average distance traveled by a moving molecule between successive collisions with other moving molecules. If the mean free path is greater than the linear size L of the chamber (which is 1 m), then it's more likely that given molecule will hit the walls of the chamber instead of the other molecule, and vice versa.

According to the kinetic theory, the mean free path l (m) of the gas molecules can be calculated as:

$$l = \frac{k_B \cdot T}{\pi\sqrt{2} \cdot P \cdot d^2},$$

where $k_B = 1.38 \cdot 10^{-23}$ J/K is the Boltzmann constant;
T is the absolute temperature, K;
P is the pressure of the gas, Pa,
and d is the effective diameter of the gas molecules, m.

Effective diameter of the nitrogen molecules is $3.74 \cdot 10^{-10}$ m.

Then $l = \frac{1.38 \cdot 10^{-23} \cdot 300}{3.14 \cdot \sqrt{2} \cdot 1 \cdot 10^{-6} \cdot (3.74 \cdot 10^{-10})^2} = 6.67 \cdot 10^3$ m.

As you see, $l >> L$, and we can make a conclusion that it's much more likely that given molecule will hit the walls of the chamber instead of the other molecule.